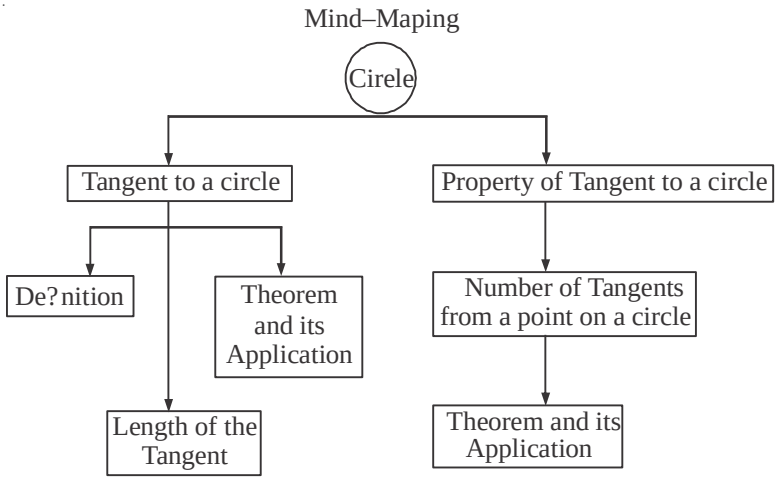
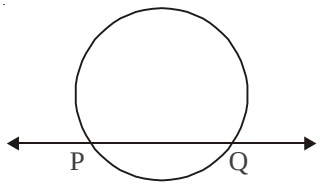
CHAPTER

# 10 Circles



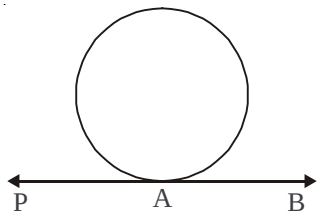

## KEY POINTS

1. A **circle** is a collection of all those points in a plane which are at a constant distance from a fixed point. The fixed point is called the **centre** and fixed distance is called the **radius**.
2. **Secant:** A line which intesects a circle in two distinct points is called a secant of the circle.




3. **Tangent**: It is a line that intersects the circle at only one point. The point where tangent touches the circle is called the point of contact.

   Here A is the poin of contact.




4. **Number of Tangent**: Infinitely many tangents can be drawn on a circle.

**5.** **Number of Secant**: There are infinitely many secants which can be drawn on a circle.

**6.** The proofs of the following theorems can be asked in the examination:–

(i) The tangent at any point of a circle is perpendicular to the radius through the point of contact.

(ii) The lengths of tangents drawn from an external point to a circle are equal.

**7.** There is only one tangent at a point of the circle.

**8.** The tangent to a circle is a special case of the secant.

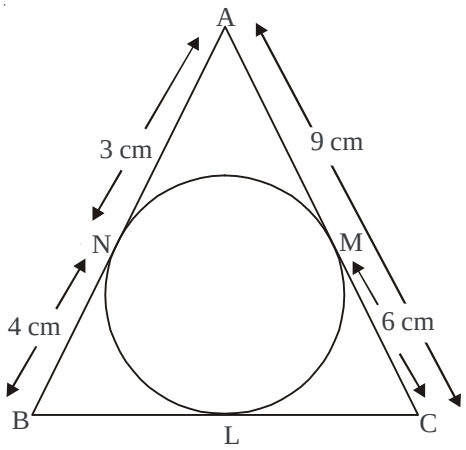
**9.** There is no tangent to a circle passing through a point lying inside the circle.

**10.** There is one and only one tangent to a circle passing through a point lying on the circle.

**11.** There are exactly two tangents to a circle through a point lying outside the circles.

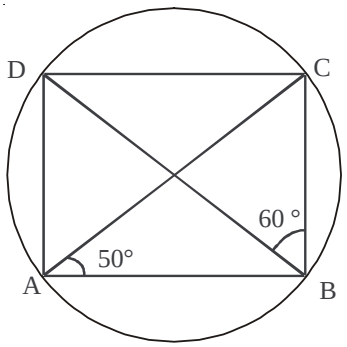
## VERY SHORT ANSWER TYPE QUESTIONS

**1.** In fig., ΔABC is circumscribing a circle. Find the length of BC.

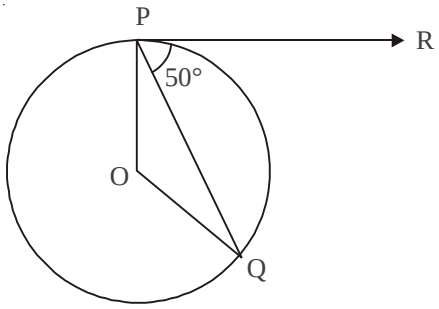


**2.** The length of the tangent to a circle from a point P, which is 25 cm away from the centre, is 24 cm. What is the radius of the circle.

**3.** In fig., ABCD is a cyclic quadrilatreral. If $\angle BAC = 50°$ and $\angle DBC = 60°$, then find $\angle BCD$.



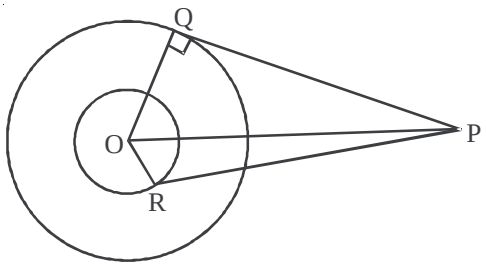

**4.** In figure, O is the centre of a circle, PQ is a chord and the tangent PR at P makes an angles of 50° with PQ. Find $\angle POQ$.



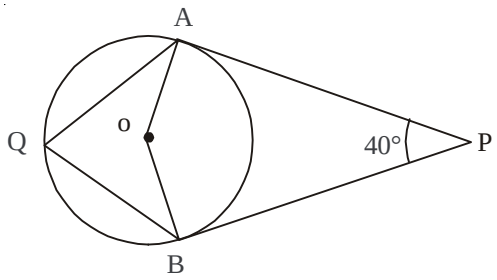

**5.** If two tangents inclined at an angle 60° are drawn to a circle of radius 3 cm, then find the length of each tangent.

**6.** If radii of two concentric circles are 4 cm and 5 cm, then find the length of the chord of one circle which is tangent to the other circle.

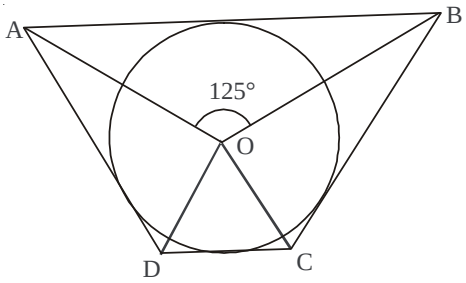
**7.** In the given figure, PQ is tangent to outer circle and PR is tangent to inner circle. If PQ = 4cm, OQ = 3 cm and QR = 2 cm then find the length of PR.



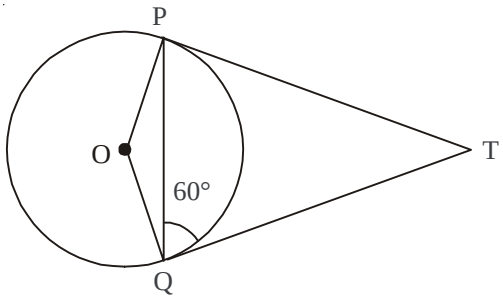
**8.** In the given figure, O is the centre of the circle, PA and PB are tangents to the circle then find ∠AQB. (CBSE 2016)




**9.** In the given figure, If ∠AOB = 125° then find ∠COD.



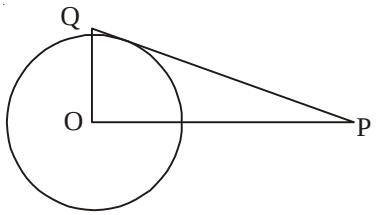

**10.** If two tangent TP and TQ are drawn from an external point T such that ∠TQP = 60° then find ∠OPQ.




**11.** How many tangents can a circle have? (NCERT)

**12.** A tangent to a circle intersects it in _____ points. (NCERT)

**13.**




If PQ is a tangent then find the value of $\angle POQ + \angle QPO$.

**14.** Choose the correct Answer.

A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is :

(a) 12 cm (b) 13 cm (c) 8.5 cm (d) $\sqrt{119}$ cm (NCERT)

**15.** A circle can have _____ parallel tangents at the most. (NCERT)

**16.** The common point of a tangent to a circle and the circle is called _____ . (NCERT)

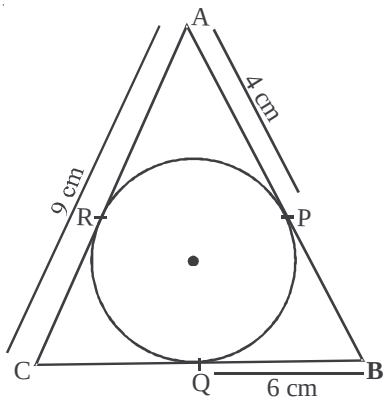
## SHORT ANSWER TYPE-I QUESTIONS

**17.** If diameters of two concentric circle are $d_1$ and $d_2$ ($d_2 > d_1$) and $c$ is the length of chord of bigger circle which is tangent to the smaller circle. Show that $d_2^2 = c^2 + d_1^2$.

**18.** The length of tangent to a circle of radius 2.5 cm from an external point P is 6 cm. Find the distance of P from the nearest point of the circle.

**19.** TP and TQ are the tangents from the external point T of a circle with centre O. If $\angle OPQ = 30°$ then find the measure of $\angle TQP$.

**20.** In the given fig. AP = 4 cm, BQ = 6 cm and AC = 9 cm. Find the semi perimeter of $\Delta ABC$.

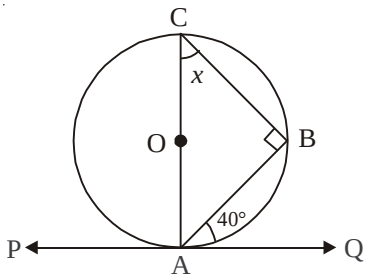



**21.** A circle is drawn inside a right angle triangle whose sides are $a$, $b$, $c$ where $c$ is the hypotenuse, which touches all the sides of the triangle. Prove $r = \frac{a + b - c}{2}$ where $r$ is the radius of the circle. (NCERT Exampler, 2012)

**22.** Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

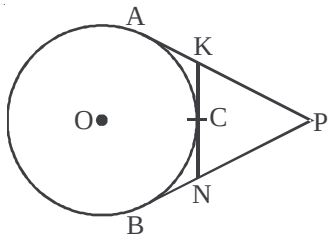
**23.** Prove that in two concentric circles the chord of the larger circle which is tangent to the smaller circle is bisected at the point of contact.

**24.** In the given Fig., AC is diameter of the circle with centre O and A is point of contact, then find *x*.

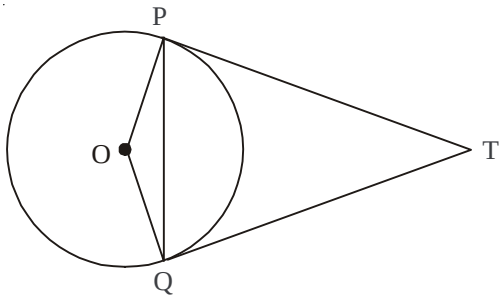



**25.** In the given fig. KN, PA and PB are tangents to the circle. Prove that:

KN = AK + BN.

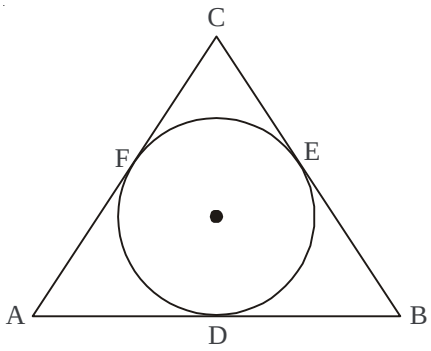



**26.** In the given fig. PQ is a chord of length 6 cm and the radius of the circle is 6 cm. TP and TQ are two tangents drawn from an external point T. Find ∠PTQ.

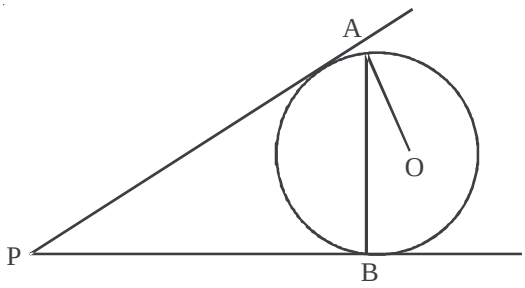


## SHORT ANSWER TYPE-II QUESTIONS

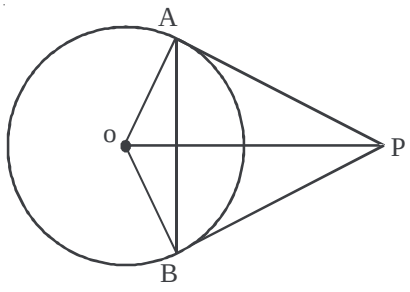
**27.** In the given figure find AD, BE, CF where AB = 12 cm, BC = 8 cm and AC = 10 cm.



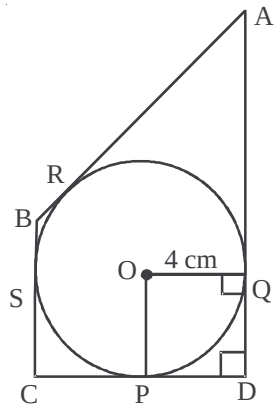

**28.** Two tangents PA and PB are drawn to a circle with centre O from an external point P. Prove that $\angle APB = 2\angle OAB$ (NCERT, Exemplar-2)




**29.** In the given fig. OP is equal to the diameter of the circle with centre O. Prove that $\Delta ABP$ is an equilateral triangle.



**30.** In the given fig., find PC. If AB = 13 cm, BC = 7 cm and AD = 15 cm.

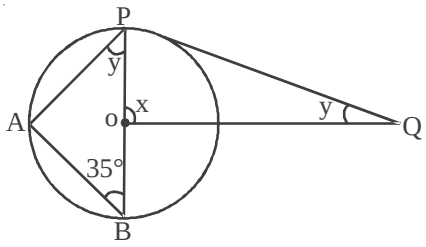



## LONG ANSWER TYPE QUESTIONS

**31.** In the given fig. find the radius of the circle.

**m**

**32.** In the given fig. PQ is tangent and PB is diameter. Find the value of $x$ and $y$.



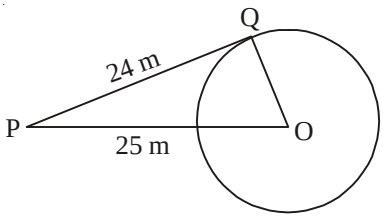

## ANSWERS AND HINTS

**1.** Since length of both the tangents from a point outside the circle is equal, So

BN = BL, CM = CL

BL + CL = BC = 10 cm

**2.**




By Pythagorous Rule, QR = 7 cm.

**3.** Angle is the same segment are euqal.

- DC is the chord so $\angle DAC = 60°$.
- The sum of the opposite angles of a cyclic quadrilateral is 180°.

So $\angle BCD = 70°$

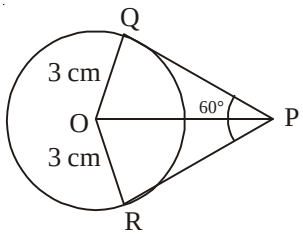
**4.** The tangent at any point of a circle is perpendicular to the radius through the point of contact.

So, $\angle RPO = 90°$

$\angle OPQ = \angle OQP = 40°$

$\angle POQ = 100°$

**5.**



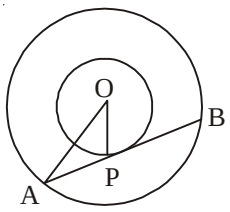

$\Delta QPO \cong \Delta RPO$

$\Rightarrow \quad \angle QPO = \angle RPO = \frac{60°}{2} = 30°$

In $\Delta QPO$, $\angle OQP = 90°$ (Tangent is perpendicular at the point of contact).

$$\tan 30° = \frac{OQ}{QP} \Rightarrow QP = 3\sqrt{3} \text{ cm}$$

**6.**




In $\Delta AOP$, right angled at P.

$$OA^2 = AP^2 + OP^2 \Rightarrow (5)^2 = AP^2 + 4^2 \Rightarrow AP^2 = 9$$

$\Rightarrow \quad AP = 3$

In $\Delta PQO$, $AB = 6$ cm

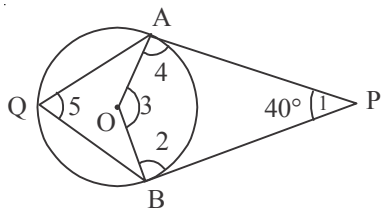
**7.** $(4)^2 + (3)^2 = (OP)^2$

$5 = OP$

In $\Delta PRO$, $(5)^2 = (2)^2 + (PR)^2$

$PR = \sqrt{21}$ cm

**8.**




In Quadrilateral PROQ

$\underline{|}1 + \underline{|}2 + \underline{|}3 + \underline{|}4 = 360°$

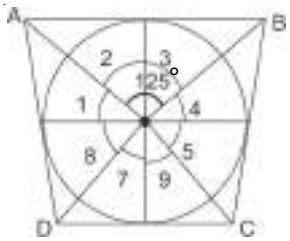
$\underline{|}1 + \underline{|}3 = 180°$

$\underline{|}3 = 140°$

Now, $\underline{|}3 = 2\underline{|}5$

$\underline{|}5 = 70°$ or $\underline{|}AQB = 70°$

**9.** $\left.\begin{array}{l}\underline{|}1 = \underline{|}2\\ \underline{|}3 = \underline{|}4\\ \underline{|}5 = \underline{|}6\\ \underline{|}7 = \underline{|}8\end{array}\right]$ (CPCT) of their corresponding triangles.




$2(\underline{|}2 + \underline{|}3 + \underline{|}6 + \underline{|}7) = 360°$

or $\underline{|}AOB + \underline{|}COD = 186°$

or $\underline{|}COD = 55°$

**10.** $\underline{|}OQT = 90°$ (Angle between tangent & radius)

$\underline{|}PQO = 30°$ $(90° - 60°)$

$\underline{|}PQO = \underline{|}OPQ = 30°$

**11.** Infinity many

**12.** One

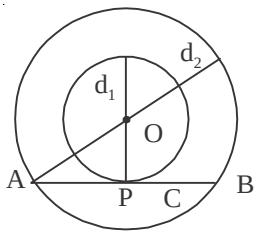
**13.** 90° as $\underline{|}OQP = 90°$ (Angle between tangent and radius of the circle)

**14.** D($\sqrt{119}$ cm)

**15.** Two

**16.** Point of Contact

**17.**




$$AO^2 = OP^2 + AP^2$$

$$\left(\frac{d_2}{2}\right)^2 = \left(\frac{d_1}{2}\right)^2 + AP^2$$

$$\left(\frac{d_2}{2}\right)^2 - \left(\frac{d_1}{2}\right)^2 = AP^2$$

$$\sqrt{\frac{1}{4}\left[(d_2)^2 - (d_1)^2\right]} = AP$$

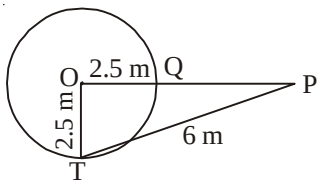
$$2\sqrt{\frac{1}{4}\left[(d_2)^2 - (d_1)^2\right]} = AB$$

$$\sqrt{(d_2)^2 - (d_1)^2} = C$$

$$(d_2)^2 - (d_1)^2 = C^2$$

$$d_2{}^2 = C^2 - d_1{}^2$$

**18.**




$$(OP)^2 = (OT)^2 + (PT)^2$$

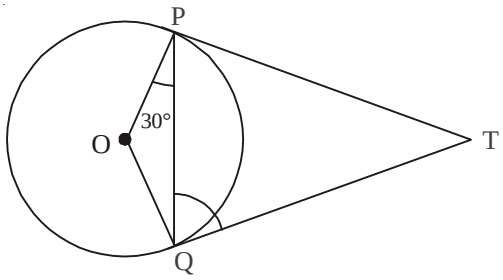
$$(OP)^2 = (2.5)^2 + (6)^2$$

$$= 42.25$$

$$(OP)^2 = (6.5)^2$$

$$QP = 4 \text{ cm}$$

**19.**




$\angle OQP = \angle OPQ = 30°$

$\angle OQT = 90°$ (Angle between radius tangent)

$\angle TQP = \angle OQT - \angle OQP$

$= 90° - 30° = 60°$

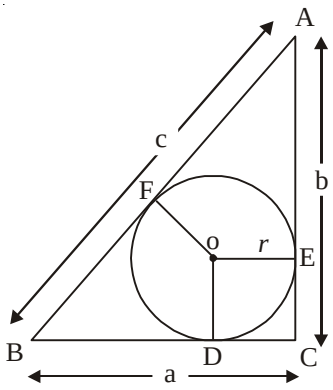
**20.** $AP = AR = 4$ cm

$CR = CQ = (9 - 4)$ cm $= 5$ cm

$= \frac{1}{2}[AC + AB + BC]$

$= \frac{1}{2}[9 + 10 + 11] = 15$ cm

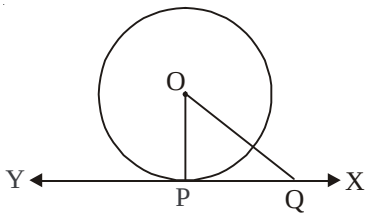
**21.**




$b - r = AF, \quad a - r = BF$

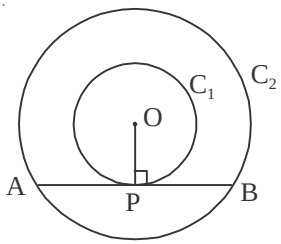
or, $AB = C = AF + BF = b - r + a - r$

This gives, $r = \frac{a + b - c}{2}$

**22.** 


(Theorem 10.1, NCERT)

**23.** Join OP




AB is tangent to $C_1$ at P and OP is radius

$OP \perp AB$

AB is chord of circle $C_2$ and $OP \perp AB$.

Therefore OP is the bisector of the chord AB as the perpendicular from the centre bisects the chord i.e,

$AP = BP$

**24.** $\angle OAB = 50°$

$x + \angle B + \angle OAB = 180°$

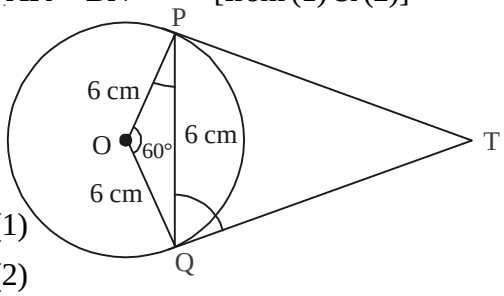
$x + 90° + 50° = 180°$

$x = 40°$

**25.** $AK = KC$ …(1)

$BN = NC$ …(2)

$KN = KC + NC = AK + BN$ [from (1) & (2)]

**26.** 


$\angle POQ + \angle PTQ = 180°$

$60° + \angle PTQ = 180°$

$\angle PTQ = 120°$

**27.** $AC = AF + FC = 10\text{ cm}$ …(1)

$AB = AD + DB = 12\text{ cm}$ …(2)

$BC = BE + CE = 8\text{ cm}$ …(3)

$$\begin{bmatrix} BD = BE \\ AD = AF \\ CF = CE \end{bmatrix} \quad \ldots(4)$$

2AD, 2FC, 2BD are obtained

Replace from (4) in (1), (2), (3) (So that in (5) + (6) + (7)). 2AD, 2FC, 2BD are obtained.

$AC = AD + FC = 10\,cm$ …(1)

$AB = AD + DB = 12\,cm$ …(2)

$BC = BD + CE = 8\,cm$ …(3)

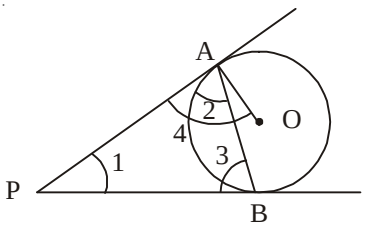
Add (5, 6, 7)

$2(AD + FC + DB) = 30$

$AD + FC + DB = 15$

Substitute values from (1), (2) & (3)
and find. AD = 7 cm, BE = 5 cm, CF = 3 cm.

**28.** $PA = PB$




So, $\angle 2 = \angle 3 = \frac{1}{2}(180° - \angle 1)$

$\angle 2 = \angle 3 = 90° - \frac{1}{2}\angle 1$

$\angle 4 = 90°$ (Angle between tangent & Radius)

$\angle OAB = \angle 4 - \angle 2$

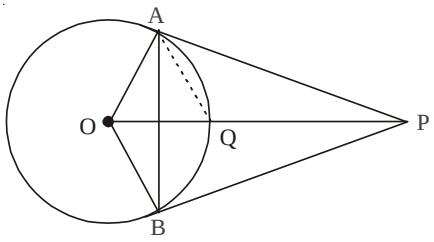
$= 90° - \left(90° - \frac{1}{2}\angle 1\right) = 90° - 90° + \frac{1}{2}\angle 1$

$\angle OAB = \frac{1}{2}\angle APB$

$2\angle OAB = \angle APB$

**29.** $OP = 2r$

$\Rightarrow$ $QP = QP = r$




Consider $\Delta AOP$ is which $OA \perp AP$ and OP is the hypotenuse.

$OQ = AQ = OA$

(Mid point of hy potenuse is equidistance from the vertices).

$\Rightarrow$ OAQ is an equilitateral triangle.

$\Rightarrow$ $\angle AOQ = 60°$

Consider right angled triangle OAP

$\angle AOQ = 60°$

$$\angle OAP = 90° \Rightarrow \angle APO = 30°$$

$$\angle APB = 2\angle APO = 2 \times 30° = 60°$$

$$PA = PB \text{ (tangents)}$$

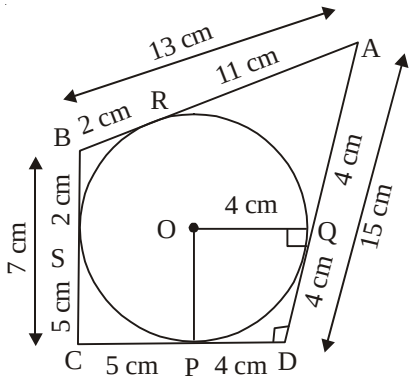
$$\Rightarrow \quad \angle PAB = \angle PBA$$

$$\text{In } \Delta APB = 60°$$

$$\angle PAB = \angle PBA = \frac{180° - 60°}{2} = 60°$$

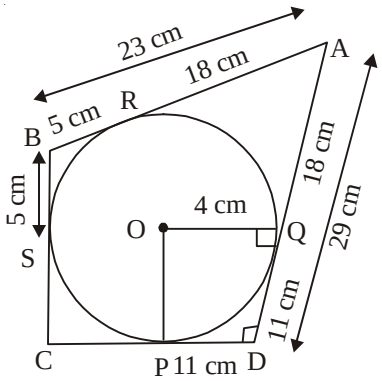
$\Rightarrow$ each angle of DPAB = 60°. **Hence Proved.**

**30.**



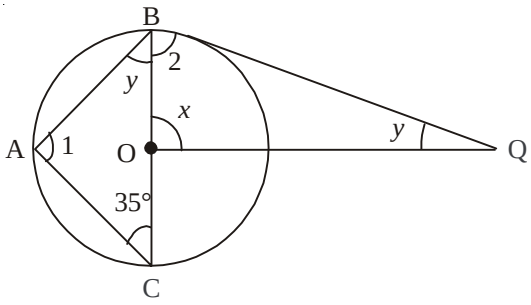

**PC or CP = 5 cm**

**31.**




**r = 11 cm**

**32.**




In $\Delta ABC$, $\angle 1 = 90°$ (Angle in semi-circles)

$\angle 1 + 35° + y = 180°$

$90° + 35° + y = 180°$

$y = 55°$

$\Delta OBQ$, $\angle 2 = 90°$ (Angle between tangent and radius)

$\angle 2 + \angle x + \angle y = 180°$

$90° + \angle x + 55° = 180°$

$x = 35°$

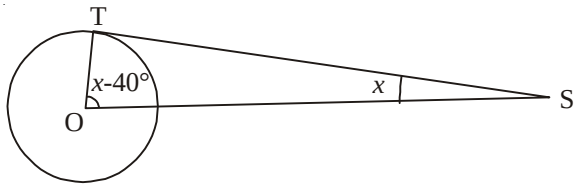
# PRACTICE-TEST

## CIRCLES

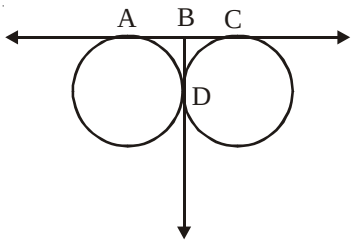
***Time : 1 Hr.*** ***M.M.: 20***

### SECTION-A

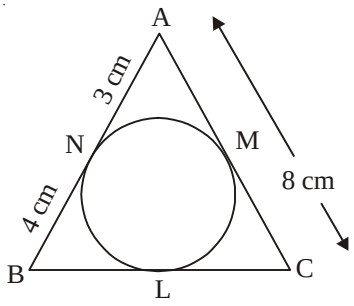
1. In the given figure find $x$, where ST is the tangent. 1




2. In the given figure if AC = 9, find BD. 1




3. In the given figure, ΔABC is circumscribing a circle, then find the length of BC. 1

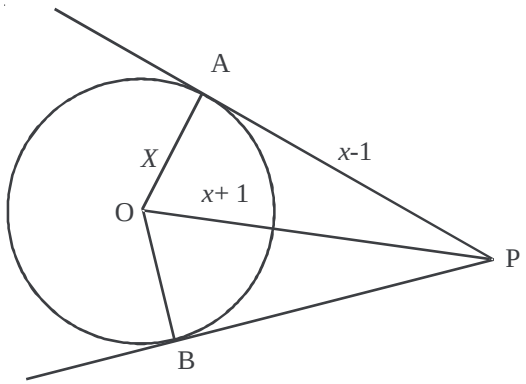



4. From the external point P tangents PA and PB are drawn to a circle with centre O. If ∠PAB = 50°, then find ∠AOB. (Delhi-2016, CBSE) 1

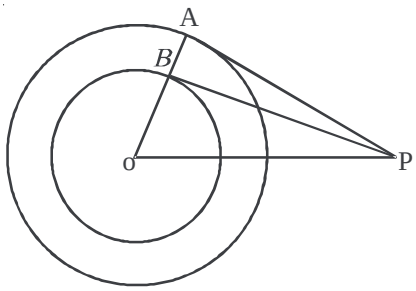
### SECTION-B

5. If the angle between two tangents drawn from an external point P to a circle of radius a and centre O is 60° then find the length of OP. (All India 2017) 2

6. In the following figure find $x$. 2

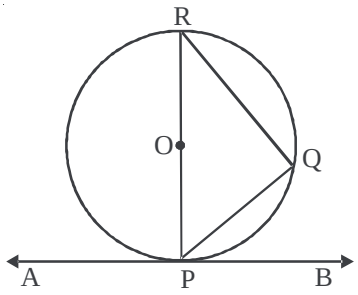



7. Two concentric circle with centre O are of radii 6 cm and 3 cm. From an external point P, tangents PA and PB are drawn to these circle as shown in the figure. If AP = 10 cm. Find BP 2



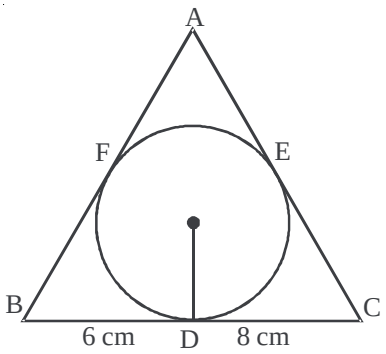

## SECTION-C

8. In the given figure, AB is a tangent to a circle with centre O. Prove ∠BPQ = ∠PRQ. 3



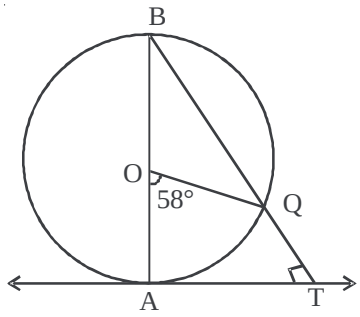

9. In the given figure ΔABC is drawn to circumscribe a circle of radius 3 cm, such that the segment BD and DC into which BC is divided by the point of

contact D are of length 6 cm and 8 cm respectively, find side AB if the *ar*(ΔABC) = 63 cm² 3




## SECTION-D

10. AB is a diameter of a circle with centre O and AT is a tangent. If ∠AOQ = 58° find ∠ATQ. 4




❐❐❐