CHAPTER

2 Polynomials

KEY POINTS

1. **Polynomial :** If x is a variable, n is a natural number and $a_0, a_1, a_2, a_3, \ldots\ldots$ a_n are real numbers, then $p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots\ldots + a_1 x + a_0$, $(a_n \neq 0)$ is called a polynomial in x.
2. Polynomials of degree 1, 2 and 3 are called linear, quadratic and cubic polynomials respectively.
3. A quadratic polynomial is an algebraic expression of the form $ax^2 + bx + c$, where a, b, c are real numbers with $a \neq 0$.
4. Zeros of a polynomial $p(x)$ are precisely the x – coordinates of the points where the graph of $y = p(x)$ intersects the x–axis, *i.e.,* $x = a$ is a zero of polynomial $p(x)$ if $p(a) = 0$
5. A polynomial can have at most the same number of zeros as the degree of the polynomial.
6. (*i*) If one zero of a quadratic polynomial $p(x)$ is negative of the other, then coefficient of x is 0.

   (*ii*) If zeroes of a quadratic polynomial $p(x)$ are reciprocal of each other, then coefficient of x^2 = constant term.
7. Relationship between zeros and coefficients of a polynomial

   If α and β are zeros of $p(x) = ax^2 + bx + c$ $(a \neq 0)$, then

   Sum of zeros $= \alpha + \beta = -\dfrac{b}{a}$

   Product of zeros $= \alpha\beta = \dfrac{c}{a}$
8. If α, β are zeros of a quadratic polynomial $p(x)$, then

   $p(x) = k\,[x^2 - (\text{sum of zeros})\,x + \text{product of zeros}]$

   $\Rightarrow p(x) = k\,[x^2 - (\alpha + \beta)x + \alpha\beta\,]$; where k is any non-zero real number.
9. Graph of linear polynomial $p(x) = ax + b$ is a straight line.
10. Division Algorithm states that given any polynomials $p(x)$ and $g(x)$, there exist polynomial $q(x)$ and $r(x)$ such that:

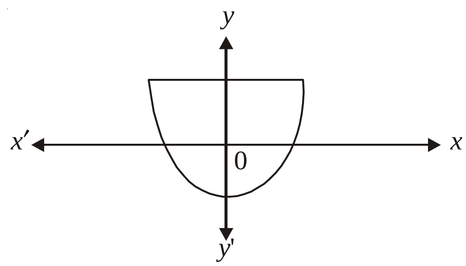
$p(x) = g(x).\ q(x) + r(x)\ ;\ g(x) \neq 0,$

[where either $r(x) = 0$ or degree $r(x) <$ degree $g(x)$]

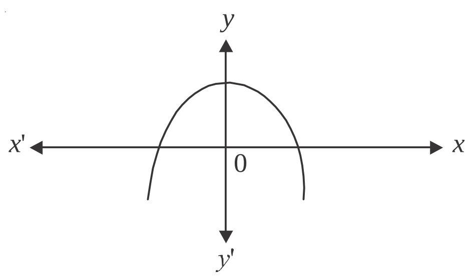
Graph of different types of polynomials:

- **Linear Polynomial :** The graph of a linear polynomial $ax + b$ is a straight line, intersecting x-axis at one point.
- **Quadratic Polynomial:**

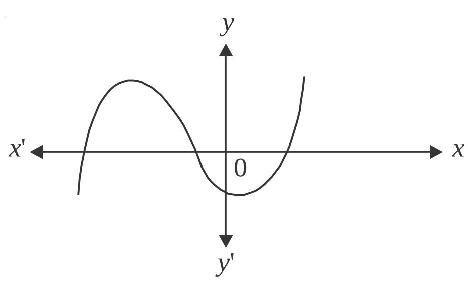
(*i*) Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola open upwards like U, if a > 0 and intersect x-axis at maximum two distinct points.



(*ii*) Graph of a quodratic polynomial $p(x) = ax^2 + bx + c$ is a parabola open downwards like $\cap$, if a < 0 and intersect x-axis at maximum two distinct points.



(*iii*) Polynomial and its graph : In general a polynomial $p(x)$ of degree n crosses the x-axis at most n points.



VERY SHORT ANSWER TYPE QUESTIONS

1. If one root of the polynomial $P(x) = 5x^2 + 13x + K$ is reciprocal of the other, then value of k is

(a) 0 (b) 5 (c) $\frac{1}{6}$ (d) 6

2. If α and β are the zeroes of the polynomial $p(x) = x^2 - p(x + 1) - c$ such that $(\alpha + 1)(\beta + 1) = 0$, the c = ________ .

3. If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is

(a) 10 (b) – 10 (c) 5 (d) – 5

4. If the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and – 3, then

(a) $a = -7, b = -1$ (b) $a = 5, b = -1$

(c) $a = 2, b = -6$ (d) $a = 0, b = -6$

5. What should be added to the polynomial $x^2 - 5x + 4$, so that 3 is the zero of the resulting polynomial:

(a) 1 (b) 2 (c) 4 (d) 5

6. If α and β are the roots of the polynomial

$$f(x) = x^2 + x + 1, \quad \text{then} \quad \frac{1}{\alpha} + \frac{1}{\beta} =$$

7. If a quadratic polynomial $f(x)$ is not factorizable into linear factors, then it has no real zero. (True/False)

8. If a quadratic polynomial $f(x)$ is a square of a linear polynomial, then its two zeros are coincident. (True/False).

9. The product of the zeros of $x^3 + 4x^2 + x - 6$ is

(a) – 4 (b) 4 (c) 6 (d) 6

10. Given that two of the zeros of the cubic polynomial $ax^3 + bx^2 + cx + d$ are 0, the third zero is

(a) $-\frac{b}{a}$ (b) $\frac{b}{a}$ (c) $\frac{c}{a}$ (d) $-\frac{d}{a}$

11. What will be the number of zeros of a linear polynomial $p(x)$ if its graph (*i*) passes through the origin. (*ii*) doesn't intersect or touch x-axis at any point?

12. Find the quadratic polynomial whose zeros are

$\left(5+2\sqrt{3}\right)$ and $\left(5-2\sqrt{3}\right)$

13. If one zero of $p(x) = 4x^2 - (8k^2 - 40k)\,x - 9$ is negative of the other, find values of k.

14. What number should be added to the polynomial $x^2 - 5x + 4$, so that 3 is a zero of polynomial so obtained.

15. How many (*i*) maximum (*ii*) minimum number of zeroes can a quadratic polynomial have?

16. What will be the number of real zeros of the polynomial $x^2 + 1$?

17. If α and β are zeros of polynomial $6x^2 - 7x - 3$, then form a quadratic polynomial where zeros are 2α and 2β (CBSE)

18. If α and $\frac{1}{\alpha}$ are zeros of $4x^2 - 17x + k - 4$, find the value of k.

19. What will be the number of zeros of the polynomials whose graphs are parallel to (*i*) y-axis (ii) x-axis?

20. What will be number of zeros of the polynomials whose graphs are either touching or intersecting the axis only at the points:

(*i*) (–3, 0), (0, 2) & (3, 0) (*ii*) (0, 4), (0, 0) and (0, –4)

SHORT ANSWER TYPE (I) QUESTIONS

21. If –3 is one of the zeros of the polynomial $(k - 1)x^2 + k\,x + 1$, find the value of k.

22. If the product of zeros of $ax^2 - 6x - 6$ is 4, find the value of a. Hence find the sum of its zeros.

23. If zeros of $x^2 - kx + 6$ are in the ratio 3 : 2, find k.

24. If one zero of the quadratic polynomial $(k^2 + k)x^2 + 68x + 6k$ is reciprocal of the other, find k.

25. If α and β are the zeros of the polynomial $x^2 - 5x + m$ such that $\alpha - \beta = 1$, find m. (CBSE)

26. If the sum of squares of zeros of the polynomial $x^2 - 8x + k$ is 40, find the value of k.

27. If α and β are zeros of the polynomial $t^2 - t - 4$, form a quadratic polynomial whose zeros are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

28. What should be added to the polynomial $x^3 - 3x^2 + 6x - 15$, so that it is completely divisible by $x - 3$? (CBSE 2016)

29. If m and n are the zeros of the polynomial $3x^2 + 11x - 4$, find the value of $\frac{m}{n} + \frac{n}{m}$.

(CBSE, 2012)

30. Find a quadratic polynomial whose zeros are $\frac{3+\sqrt{5}}{5}$ and $\frac{3-\sqrt{5}}{5}$.

(CBSE, 2013)

SHORT ANSWER TYPE (II) QUESTIONS

31. If $(k + y)$ is a factor of each of the polynomials $y^2 + 2y - 15$ and $y^3 + a$, find the values of k and a.

32. Obtain zeros of $4\sqrt{3}\,x^2 + 5x - 2\sqrt{3}$ and verify relation between its zeroes and coefficients.

33. If $x^4 + 2x^3 + 8x^2 + 12x + 18$ is divided by $(x^2 + 5)$, remainder comes out to be $(px + q)$, find values of p and q.

34. –5 is one of the zeros of $2x^2 + px - 15$, zeroes of $p(x^2 + x) + k$ are equal to each other. Find the value of k.

35. Find the value of k such that $3x^2 + 2kx + x - k - 5$ has the sum of zeros as half of their product.

36. If α and β are zeros of $y^2 + 5y + m$, find the value of m such that $(\alpha + \beta)^2 - \alpha\beta = 24$

37. If α and β are zeros of $x^2 - x - 2$, find a polynomial whose zeros are $(2\alpha + 1)$ and $(2\beta + 1)$

38. Find values of a and b so that $x^4 + x^3 + 8x^2 + ax + b$ is divisible by $x^2 + 1$.

39. What must be subtracted from $8x^4 + 14x^3 - 2x^2 + 7x - 8$ so that the resulting polynomial is exactly divisible by $4x^2 + 3x - 2$?

40. What must be added to $4x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is divisible by $x^2 - 2x - 3$?

LONG ANSWER TYPE QUESTIONS

41. Find all zeros of the polynomial $2x^3 + x^2 - 6x - 3$ if two of its zeroes are $\sqrt{3}$ and $-\sqrt{3}$.

42. If $\sqrt{2}$ is a zero of $(6x^3 + \sqrt{2}\,x^2 - 10x - 4\sqrt{2})$, find its other zeroes.

43. If two zeros of $x^4 - 6x^3 - 26x^2 + 138\,x - 35$ are $(2 \pm \sqrt{3})$, find other zeroes.

44. On dividing the polynomial $x^3 - 5x^2 + 6x - 4$ by a polynomial $g(x)$, quotient and remainder are $(x - 3)$ and $(-3x + 5)$ respectively. Find $g(x)$

45. Obtain all zeros of the polynomial $2x^4 - 2x^3 - 7x^2 + 3x + 6$ if two factors of this polynomial are $\left(x \pm \sqrt{\frac{3}{2}}\right)$.

46. If the polynomial $x^4 - 3x^3 - 6x^2 + kx - 16$ is exactly divisible by $x^2 - 3x + 2$, then find the value of k. (CBSE, 2014)

47. If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by $x^2 - 2x + k$, then find the vlaue of k and a. (CBSE)

48. If α and β are zeros of the polynomial $x^2 + 4x + 3$, find the polynomial whose zeros are $1 + \frac{\beta}{\alpha}$ and $1 + \frac{\alpha}{\beta}$. (CBSE)

49. Find K, so that $x^2 + 2x + K$ is a factor of $2x^4 + x^3 - 14x^2 + 5x + 6$. Also find all the zeros of the two polynomials: (Exempler, HOTS)

50. If $x - \sqrt{5}$ is a factor of the cubic polynomial $x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}$, then find all the zeros of the polynomial.

ANSWERS AND HINTS

1. (b) 5

2. –1

3. (b) –10

4. (d) a = 0, b = –6

5. (b) 2

6. – 1

7. True

8. True

9. (c) 6

10. (a) $-\frac{b}{a}$

11. (i) 1 (ii) 0

12. $x^2 - 10x + 13$

13. $k = 0, 5$

14. 2

15. (i) 2 (ii) 0

16. 0

17. $3x^2 - 7x - 6$

18. $k = 8$

19. (i) 1 (ii) 0

20. (i) 2 (ii) 1

21. 4/3

22. $a = -\frac{3}{2}$, sum of zeroes = – 4

23. – 5, 5

24. 5

25. 6

26. 12

27. $4t^2 + t - 1$

28. On dividing $x^3 - 3x^2 + 6x - 15$ by $x - 3$, remainder is + 3, hence – 3 must be added to $x^3 - 3x^2 + 6x - 15$.

29. $$\frac{m}{n} + \frac{n}{m} = \frac{m^2 + n^2}{mn} = \frac{(m+n)^2 - 2mn}{mn} = \frac{\left(-\frac{11}{3}\right)^2 - 2\left(-\frac{4}{3}\right)}{-\frac{4}{3}} = -\frac{145}{12}$$

30. $\alpha + \beta = \frac{6}{5}, \quad \alpha\beta = \frac{4}{25},$
$25x^2 - 30x + 4$

31. $k = 3, -5$ and $a = 27, -125$

32. $-\frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$

33. $p = 2, \; q = 3$

34. $\frac{7}{4}$

35. 1

36. 1

37. $x^2 - 4x - 5$

38. $a = 1, b = 7$

39. $14x - 10$

40. $61x - 65$

41. $\sqrt{3}, \; -\sqrt{3}, \; -\frac{1}{2}$

42. $-\frac{\sqrt{2}}{2}, \frac{-2\sqrt{2}}{3}$

43. – 5, 7

44. $x^2 - 2x + 3$

45. $2, \ -1, \ \mp\sqrt{\frac{3}{2}}$

46. $x^2 - 3x + 2 = (x - 2)(x - 1)$

$P(1) = 0, K = 24.$

47. On dividing $x^4 - 6x^3 + 16x^2 - 25x + 10$ by $x^2 - 2x + k$ we get remainder $(2k - 9)x + (10 - 8k + k^2)$

Given remainder = $x + 9$

$$2k - 9 = 1 \quad \Rightarrow \quad k = 5$$

$$10 - 8k + k^2 = a \quad \Rightarrow \quad a = 10 - 40 + 25 = -5$$

$$a = -5, \quad k = 5$$

48. $x^2 - \frac{16}{3}x + \frac{16}{3}$ or $\frac{1}{3}(3x^2 - 16x + 16)$

49. On dividing $2x^4 + x^3 - 14x^2 + 5x + 6$ by $x^2 + 2x + k$

We get $(7k + 21)x + 2k^2 + 8k + 6$ as remainder is zero.

$\Rightarrow \quad 7k + 21 = 0$ and $2k^2 + 8k + 6 = 0$

$\Rightarrow \quad k = -3$ and $k = -1$ or -3

$\Rightarrow \quad k = -3$

Zeros of $x^2 + 2x - 3$ are $1, -3$ and $2x^4 + x^3 - 14x^2 + 5x + 6$ are $1, \ -3, \ 2, \ -\frac{1}{2}$

50. $\sqrt{5}, \ \sqrt{5} + \sqrt{2}, \ \sqrt{5} - \sqrt{2}$

PRACTICE-TEST

Polynomials

Time : 1 Hr. ***M.M. : 20***

SECTION-A

1. If α and β are zeros of a quadratic polynomial $p(x)$, then factorize $p(x)$. **1**

2. If α and β are zeros of $x^2 - x - 1$, find the value of $\frac{1}{\alpha}+\frac{1}{\beta}$. **1**

3. If one of the zeros of quadratic polynomial $(K-1)x^2 + kx + 1$ is -3 then the value of K is, **1**

(a) $\frac{4}{3}$ (b) $-\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $-\frac{2}{3}$

4. A quadratic polynomial, whose zeros are -3 and 4, is **1**

(a) $x^2 - x + 12$ (b) $x^2 + x + 12$

(c) $\frac{x^2}{2} - \frac{x}{2} - 6$ (d) $2x^2 + 2x - 24$

SECTION-B

5. If α and β are zeros of $x^2 - (k+6)x + 2(2k-1)$. find the value of k if $\alpha + \beta = \frac{1}{2}\alpha\beta$. **2**

6. Find a quadratic polynomial one of whose zeros is $(3+\sqrt{2})$ and the sum of its zeroes is 6. **2**

7. If zeros of the polynomial $x^2 + 4x + 2a$ are α and $\frac{2}{\alpha}$ then find the value of a. **2**

SECTION-C

8. Find values of a and b if $(x^2 + 1)$ is a factor of the polynomial $x^4 + x^3 + 8x^2 + ax + b$. **3**

9. If truth and lie are zeros of the polynomial $px^2 + qx + r$, $(p \neq 0)$ and zeros are reciprocal to each other, Find the relation between p and r. **3**

SECTION-D

10. On dividing the polynomial $x^3 + 2x^2 + kx + 7$ by $(x - 3)$, remainder comes out to be 25. Find quotient and the value of k. Also find the sum and product of zeros of the quotient so obtained. **4**

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