MATHEMATICAL REASONING

14.1 Overview

If an object is either black or white, and if it is not black, then logic leads us to the conclusion that it must be white. Observe that logical reasoning from the given hypotheses can not reveal what "black" or "white" mean, or why an object can not be both. Infact, logic is the study of general patterns of reasoning, without reference to particular meaning or context.

14.1.1 *Statements*

A statement is a sentence which is either true or false, but not both simultaneously.

Note: No sentence can be called a statement if

  (i)   It is an exclamation
 (ii)   It is an order or request
(iii)   It is a question
(iv)   It involves variable time such as 'today', 'tomorrow', 'yesterday' etc.
 (v)   It involves variable places such as 'here', 'there', 'everywhere' etc.
(vi)   It involves pronouns such as 'she', 'he', 'they' etc.

Example 1

  (i)   The sentence

      'New Delhi is in India; is true. So it is a statement.

 (ii)   The sentence

      "Every rectangle is a square" is false. So it is a statement.

(iii)   The sentence

      "Close the door" can not be assigned true or false (Infact, it is a command). So it can not be called a statement.

(iv)   The sentence

"How old are you?" can not be assigned true or false (In fact, it is a question). So it is not a statement.

(v)   The truth or falsity of the sentence

"*x* is a natural number" depends on the value of *x*. So it is not considered as a statement. However, in some books it is called an open statement.

Note: Truth and falisity of a statement is called its truth value.

14.1.2 *Simple statements* A statement is called simple if it can not be broken down into two or more statements.

Example 2   The statements

"2 is an even number",

"A square has all its sides equal" and

" Chandigarh is the capital of Haryana" are all simple statements.

14.1.3 *Compound statements* A compound statement is the one which is made up of two or more simple statements.

Example 3 The statement

"11 is both an odd and prime number" can be broken into two statements

"11 is an odd number" and "11 is a prime number" so it is a compound statement.

Note: The simple statements which constitutes a compound statement are called **component statements**.

14.1.4 *Basic logical connectives*   There are many ways of combining simple statements to form new statements. The words which combine or change simple statements to form new statements or compound statements are called **Connectives**. The basic connectives (logical) *conjunction* corresponds to the English word 'and'; *disjunction* corresponds to the word 'or'; and *negation* corresponds to the word 'not'.

Throughout we use the symbol '$\wedge$' to denote conjunction; '$\vee$' to denote disjunction and the symbol '~' to denote negation.

Note: Negation is called a connective although it does not combine two or more statements. In fact, it only *modifies* a statement.

14.1.5 *Conjunction*   If two simple statements *p* and *q* are connected by the word 'and', then the resulting compound statement "*p* and *q*" is called a conjunction of *p* and *q* and is written in symbolic form as "$p \wedge q$".

Example 4 Form the conjunction of the following simple statements:

   p :   Dinesh is a boy.
   q :   Nagma is a girl.

Solution The conjunction of the statement *p* and *q* is given by

 p ∧ *q* :   Dinesh is a boy and Nagma is a girl.

Example 5 Translate the following statement into symbolic form
              "Jack and Jill went up the hill."

Solution The given statement can be rewritten as

              "Jack went up the hill and Jill went up the hill"

Let *p* : Jack went up the hill   and   *q* : Jill went up the hill.

Then the given statement in symbolic form is *p* ∧ *q*.

Regarding the truth value of the conjunction *p* ∧ *q* of two simple statements *p* and *q*, we have

 (D₁) :   *The statement p ∧ q has the truth value T (true) whenever both p and q have the truth value T.*

 (D₂) :   *The statement p ∧ q has the truth value F (false) whenever either p or q or both have the truth value F.*

Example 6 Write the truth value of each of the following four statements:

  (i)   Delhi is in India and 2 + 3 = 6.
  (ii)   Delhi is in India and 2 + 3 = 5.
  (iii)   Delhi is in Nepal and 2 + 3 = 5.
  (iv)   Delhi is in Nepal and 2 + 3 = 6.

Solution In view of (D₁) and (D₂) above, we observe that statement (i) has the truth value F as the truth value of the statement "2 + 3 = 6" is F. Also, statement (ii) has the truth value T as both the statement "Delhi is in India" and "2 + 3 = 5" has the truth value T.

Similarly, the truth value of both the statements (iii) and (iv) is F.

14.1.6 *Disjunction*   If two simple statements *p* and *q* are connected by the word 'or', then the resulting compound statement "*p* or *q*" is called *disjunction* of *p* and *q* and is written in symbolic form as "*p* ∨ *q*".

Example 7 Form the disjunction of the following simple statements:

   p :   The sun shines.
   q :   It rains.

Solution The disjunction of the statements p and q is given by

$p \vee q$:   The sun shines or it rains.

Regarding the truth value of the disjunction $p \vee q$ of two simple statements p and q, we have

(D_3) :   *The statement $p \vee q$ has the truth value F whenever both p and q have the truth value F.*

(D_4) :   *The statement $p \vee q$ has the truth value T whenever either p or q or both have the truth value T.*

Example 8 Write the truth value of each of the following statements:

  (i)   India is in Asia or $2 + 2 = 4$.

 (ii)   India is in Asia or $2 + 2 = 5$.

(iii)   India is in Europe or $2 + 2 = 4$.

(iv)   India is in Europe or $2 + 2 = 5$.

Solution In view of (D_3) and (D_4) above, we observe that only the last statement has the truth value F as both the sub-statements "India is in Europe" and "$2 + 2 = 5$" have the truth value F. The remaining statements (i) to (iii) have the truth value T as at least one of the sub-statements of these statements has the truth value T.

14.1.7 *Negation* An assertion that a statement fails or denial of a statement is called the negation of the statement. The negation of a statement is generally formed by introducing the word "not" at some proper place in the statement or by prefixing the statement with "It is not the case that" or It is false that".

The negation of a statement p in symbolic form is written as "$\sim p$".

Example 9 Write the negation of the statement

p : New Delhi is a city.

Solution The negation of p is given by

$\sim p$: New Delhi is not a city

or $\sim p$: It is not the case that New Delhi is a city.

or $\sim p$: It is false that New Delhi is a city.

Regarding the truth value of the negation $\sim p$ of a statement p, we have

(D_5) :   *$\sim p$ has truth value T whenever p has truth value F*.

(D_6) :   *$\sim p$ has truth value F whenever p has truth value T*.

Example 10 Write the truth value of the negation of each of the following statements:

(i) p : Every square is a rectangle.

(ii) q : The earth is a star.

(iii) $r : 2 + 3 < 4$

Solution In view of (D_5) and (D_6), we observe that the truth value of ~p is F as the truth value of p is T. Similarly, the truth value of both ~q and ~r is T as the truth value of both statements q and r is F.

14.1.8 *Negation of compound statements*

14.1.9 *Negation of conjunction* Recall that a conjunction $p \wedge q$ consists of two component statements p and q both of which exist simultaneously. Therefore, the negation of the conjunction would mean the negation of at least one of the two component statements. Thus, we have

(D_7) :   *The negation of a conjunction $p \wedge q$ is the disjunction of the negation of p and the negation of q. Equivalently, we write*

$$\sim (p \wedge q) = ~\sim p ~\vee \sim q$$

Example 11 Write the negation of each of the following conjunctions:

(a)   Paris is in France and London is in England.

(b) $2 + 3 = 5$ and $8 < 10$.

Solution

(a)   Write p : Paris is in France and q : London is in England.

Then, the conjunction in (a) is given by $p \wedge q$.

Now   ~ p : Paris is not in France, and

~ q : London is not in England.

Therefore, using (D_7), negation of $p \wedge q$ is given by

~ ($p \wedge q$) = Paris is not in France or London is not in England.

(b)   Write $p : 2 + 3 = 5$ and $q : 8 < 10$.

Then the conjunction in (b) is given by $p \wedge q$.

Now ~ $p : 2 + 3 \neq 5$ and ~ $q : 8 \not< 10$.

Then, using (D_7), negation of $p \wedge q$ is given by

$- (p \wedge q) = (2 + 3 \neq 5$) or $(8 \not< 10)$

14.1.10 *Negation of disjunction* Recall that a disjunction $p \vee q$ is consisting of two component statements p and q which are such that either p or q or both exist. Therefore, the negation of the disjunction would mean the negation of both p and q simultaneously. Thus, in symbolic form, we have

(D_8) : *The negation of a disjunction $p \vee q$ is the conjunction of the negation of p and the negation of q. Equivalently, we write*

$$\sim (p \vee q) = \; \sim p \wedge \sim q$$

Example 12 Write the negation of each of the following disjunction :

       (a)   Ram is in Class X or Rahim is in Class XII.

       (b)   7 is greater than 4 or 6 is less than 7.

Solution

(a)   Let p : Ram is in Class X and q : Rahim is in Class XII.

             Then the disjunction in (a) is given by $p \vee q$.

     Now $\sim p$:   Ram is not in Class X.

           $\sim q$:   Rahim is not in Class XII.

Then, using (D_8), negation of $p \vee q$ is given by

       $\sim (p \vee q)$: Ram is not in Class X and Rahim is not in Class XII.

(b)   Write p : 7 is greater than 4, and q : 6 is less than 7.

Then, using (D_8), negation of $p \vee q$ is given by

       $\sim (p \vee q)$: 7 is not greater than 4 and 6 is not less than 7.

14.1.11 *Negation of a negation* As already remarked the negation is not a connective but a modifier. It only modifies a given statement and applies only to a single simple statement. Therefore, in view of (D_5) and (D_6), for a statement p, we have

(D_9) : *Negation of negation of a statement is the statement itself. Equivalently, we write*

$$\sim (\sim p) = p$$

14.1.12 *The conditional statement* Recall that if p and q are any two statements, then the compound statement "**if p then q**" formed by joining p and q by a connective **'if then'** is called a *conditional statement* or an *implication* and is written in symbolic form as $p \rightarrow q$ or $p \Rightarrow q$. Here, p is called hypothesis (or antecedent) and q is called *conclusion* (or *consequent*) of the conditional statement $(p \Rightarrow q)$:

Remark The conditional statement $p \Rightarrow q$ can be expressed in several different ways. Some of the common expressions are :

(a)   if p, then q

(b) q if p

(c) p only if q

(d) p is sufficient for q

(e) q is necessary for p.

Observe that the conditional statement $p \rightarrow q$ reflects the idea that whenever it is known that p is true, it will have to follow that q is also true.

Example 13 Each of the following statements is also a conditional statement.

  (i)   If $2 + 2 = 5$, then Rekha will get an ice-cream.

 (ii)   If you eat your dinner, then you will get dessert.

(iii)   If John works hard, then it will rain today.

(iv)   If ABC is a triangle, then $\angle A + \angle B + \angle C = 180°$.

Example 14 Express in English, the statement $p \rightarrow q$, where

   p : it is raining today

   $q : 2 + 3 > 4$

Solution The required conditional statement is

   "If it is raining today, then $2 + 3 > 4$"

14.1.13 *Contrapositive of a conditional statement* The statement "$(\sim q) \rightarrow (\sim p)$" is called the contrapositive of the statement $p \rightarrow q$

Example 15 Write each of the following statements in its equivalent contrapositive form:

  (i)   If my car is in the repair shop, then I cannot go to the market.

 (ii)   If Karim cannot swim to the *fort*, then he cannot swim across the river.

Solution (i) Let "p : my car is in the repair shop" and "q : I can not go to the market".

Then, the given statement in symbolic form is $p \rightarrow q$. Therefore, its contrapositive is given by $\sim q \rightarrow \sim p$.

Now $\sim p$: My car is not in the repair shop.

and $\sim q$: I can go to the market

Therefore, the contrapositive of the given statement is

   "If I can go to the market, then my car is not in the repair shop".

 (ii)   Proceeding on the lines of the solution of (i), the contrapositive of the statement in (ii) is

   "If Karim can swim across the river, then he can swim to the fort".

14.1.14 *Converse of a conditional statement* The conditional statement "$q \rightarrow p$" is called the converse of the conditional statement "$p \rightarrow q$"

Example 16 Write the converse of the following statements

(i)   If $x < y$, then $x + 5 < y + 5$

(ii)   If ABC is an equilateral triangle, then ABC is an isosceles triangle

Solution (i) Let

$p : x < y$

$q : x + 5 < y + 5$

Therefore, the converse of the statement $p \rightarrow q$ is given by

"If $x + 5 < y + 5$, then $x < y$

(ii)   Converse of the given statement is

"If ABC is an isosceles triangle, then ABC is an equilateral triangle."

14.1.15 *The biconditional statement* If two statements p and q are connected by the connective 'if and only if' then the resulting compound statement "p if and only if q" is called a *biconditional* of p and q and is written in symbolic form as $p \leftrightarrow q$.

Example 17 Form the biconditional of the following statements:

p : One is less than seven

q : Two is less than eight

Solution The biconditional of of p and q is given by "One is less than seven, if and only if two is less than eight".

Example 18 Translate the following biconditional into symbolic form:

"ABC is an equilateral triangle if and only if it is equiangular".

Solution Let p : ABC is an equilateral triangle

and q **:** ABC is an equiangular triangle.

Then, the given statement in symbolic form is given by $p \leftrightarrow q$.

14.1.16 *Quantifiers* Quantifieres are the phrases like 'These exist' and "for every". We come across many mathematical statement containing these phrases. For example – Consider the following statements

p : For every prime number x, $\sqrt{x}$ is an irrational number.

q : There exists a triangle whose all sides are equal.

14.1.17 *Validity of statements* Validity of a statement means checking when the statement is true and when it is not true. This depends upon which of the connectives, quantifiers and implication is being used in the statement.

  (i)   Validity of statement with 'AND'

        To show statement $r : p \wedge q$ is true, show statement 'p' is true and statement 'q' is true.

  (ii) Validity of statement with 'OR'

        To show statement $r : p \vee q$ is true, show either statement 'p' is true or statement 'q' is true.

  (iii) Validity of statement with "If-then"

        To show statement r : "If p then q is true", we can adopt the following methods:

        (a) Direct method : Assume p is true and show q is true, i.e., $p \Rightarrow q$.

        (b) Contrapositive method : Assume $\sim q$ is true and show $\sim p$ is true, i.e., $\sim q \Rightarrow \sim p$.

        (c) Contradiction method : Assume that p is true and q is false and obtain a contradiction from assumption.

        (d) By giving a counter example : To prove the given statement r is false we give a counter example. Consider the follwoing statement.

             "r : All prime numbers are odd". Now the statement 'r' is false as 2 is a prime number and it is an even number.

14.1.18 Validity of the statement with "If and only If" To show the statement $r : p$ if and only if q is true, we proceed as follows:

Step 1 Show if p is true then q is true.

Step 2 Show if q is true then p is true.

14.2 Solved Examples

Short Answer Type

Example 1 Which of the following statements are compound statements

  (i)   "2 is both an even number and a prime number"

  (ii) "9 is neither an even number nor a prime number"

  (iii) "Ram and Rahim are friends"

Solution

(i) The given statement can be broken into two simple statements "2 is an even number" and "2 is a prime number" and connected by the connective 'and'

(ii) The given statement can be broken into two simple statements

"9 is not an even number" and "9 is not a prime number" and connected by the connective 'and'

(iii) The given statement can not be broken into two simple statements and hence it is not a compound statement.

Example 2 Identify the component statements and the connective in the following compound statements.

(a) It is raining or the sun is shining.

(b) 2 is a positive number or a negative number.

Solution

(a) The component statements are given by

p : It is raining

q : The sun is shining

The connective is "or"

(b) The component statements are given by

p : 2 is a positive number

q : 2 is a negative number

The connective is 'or'

Example 3 Translate the following statements in symbolic form

(i) 2 and 3 are prime numbers

(ii) Tigers are found in Gir forest or Rajaji national park.

Solution

(i) The given statement can be rewritten as "2 is a prime number and 3 is a prime number".

Let   *p* : 2 is a prime number

q : 3 is a prime number

Then the given statement in symbolic form is $p \wedge q$.

(ii) The given statement can be rewritten as

"Tigers are found in Gir forest or Tigers are found in Rajaji national park"

Let p : Tigers are found in Gir forest

   q : Tigers are found in Rajaji national park.

Then the given statement in symbolic form is $p \vee q$.

Example 4 Write the truth value of each of the following statements.

 (i)   9 is an even integer or $9 + 1$ is even.

 (ii) $2 + 4 = 6$ or $2 + 4 = 7$

 (iii)   Delhi is the capital of India and Islamabad is the capital of Pakistan.

 (iv)   Every rectangle is a square and every square is a rectangle.

 (v)   The sun is a star or sun is a planet.

Solution In view of (D_1), (D_2), (D_3) and (D_4), we observe that only statement (iv) has truth value F as the first component statement namely "every rectangle is a square" is false.

Further, in statements (i), (ii) and (v) atleast one component statement is true. Therefore, these statements have truth value T.

Also, truth value of statement (iii) is T as both the component statements are true.

Example 5 Write negation of the statement

   "Everyone who lives in India is an Indian"

Solution Let p : Everyone who lives in India is an Indian. The negation of this statement is given by

   $\sim p$: It is false that everyone who lives in India is an Indian.

                    or

   $\sim p$: Everyone who lives in India is not an Indian.

Example 6 Write the negation of the following statements :

            (a) p : All triangles are equilateral triangles.

            (b) q : 9 is a multiple of 4.

            (c) r : A triangle has four sides.

Solution

 (a)   We have

   It is false that all triangles are equilateral triangles

                                      or

~ p : Threre exists a triangle which is not an equilateral triangles.

<div align="center">or</div>

~ p : Not all triangles are equilateral triangles

(b)   ~ q : 9 is not a multiple of 4.

(c)   ~ r : It is false that the triangle has four sides.

<div align="center">or</div>

~ r : A triangle has not four sides.

Example 7 Write the negation of the following statements :

(i)   Suresh lives in Bhopal or he lives in Mumbai.

(ii) $x + y = y + x$ and 29 is a prime number.

Solution

(i)   Let

p : Suresh lives in Bhopal

and q : Suresh lives in Mumbai

Then the disjunction in (i) is given by $p \vee q$.

Now   ~ p : Suresh does not live in Bhopal.

~ q : Suresh does not live in Mumbai.

Therefore, using (D$_8$), negation of $p \vee q$ is given by

~ ($p \vee q$) : Suresh does not live in Bhopal and he does not live in Mumbai.

(ii)   Let $p : x + y = y + x$

and q : 29 is a prime number.

Then the conjunction in (ii) is given by $p \wedge q$.

Now   ~ $p : x + y \neq y + x$

and   ~ q : 29 is not a prime number.

Therefore, using (D$_7$), negation of $p \wedge q$ is given by,

~ ($p \wedge q$) : $x + y \neq y + x$ or 29 is not a prime number.

Example 8 Rewrite each of the following statements in the form of conditional statements :

(i)   Mohan will be a good student if he studies hard.

(ii) Ramesh will get dessert only if he eats his dinner.

(iii) When you sing, my ears hurt.

(iv)   A necessary condition for Indian team to win a cricket match is that the selection committee selects an all-rounder.

(v)   A sufficient condition for Tara to visit New Delhi is that she goes to the Rashtrapati Bhawan.

Solution

(i)   The given statement is of the form "q if p", where

   p : Mohan studies hard.

   q : He will be a good student.

It is an equivalent form (Remark (b) 14.1.12) of the statement "if p then q". So the equivalent formulation of the given statement is

"If Mohan studies hard, then he will be a good student".

(Here, note that in p he is replaced by Mohan and in q Mohan is replaced by he)

(ii)   The given statement is of the form

   "p only if q" which is an equivalent form (Remark (c) 14.1.12) of the statement "if p then q". So, the equivalent formulation of the given statement is:

   "If Ramesh eats his dinner, then he will get dessert"

(iii)   Here 'when' means the same as 'if' and so the equivalent formulation of the given statements is:

   "If you sing, then my ears hurt"

(iv)   The given statement is of the form "q is necessary for p" where

   p : Indian team wins a cricket match

   q : The selection committee selects an all-rounder

   which is an equivalent form (Remark (e) 14.1.12) of "if p then q". So the equivalent formulation of the given statement is

   "If the teams wins a cricket match then selection committee selects an all rounder.

(v)   The given statement is of the form "p is sufficient for q" where

   p : Tara goes to Rashtrapati Bhawan

   q : She visits New Delhi

   which is an equivalent form (Remark (d) 14.1.12) of "if p, then q", so the equivalent formulation of the given statement is

   "If Tara goes to Rashtrapati Bhawan, then she visits New Delhi".

Example 9 Express in English, the statement $p \rightarrow q$, where

   p : It is raining today

   q : $2 + 3 > 4$

Solution The conditional statement is

   "If it is raining today, then $2 + 3 > 4$".

Example 10 Translate the following statements in symbolic form:

   If $x = 7$ and $y = 4$" then $x + y = 11$.

Solution Let p : $x = 7$ and $y = 4$ and q : $x + y = 11$

Then the given statement is symbolic form is $p \rightarrow q$

Example 11 Form the biconditional of the following statements :

   p : Today is 14ᵗʰ of August

   q : Tomorrow is Independence day

Solution The biconditional $p \leftrightarrow q$ is given by

   "Today is 14ᵗʰ of August if and only if tomorrow is Independence Day".

Example 12 Translate the following biconditional into symbolic form:

"ABC is an equilateral triangle if and only if its each interior angle is 60°"

Solution Let p : ABC is an equilateral triangle

 and q : Each interior angle of triangle ABC is 60°

Then the given statement in symbolic form is $p \leftrightarrow q$.

Example 13 Identify the quantifiers and write the negation of the following statements

   (i)   There exists a number which is equal to its square.

  (ii)   For all even integers x, x^2 is also even.

 (iii)   There exists a number which is a multiple of 6 and 9.

Solution (i) The quantifier is "there exists" and the negation is

"There does not exist a number which is equal to its square"

  (ii)   The quantifier is "for all" and the negation is

   "There exists an even integer x such that x^2 is not even"

 (iii)   The quantifier is "there exists" and the negation is

   "There does not exist a number which is a multiple of both 6 and 9".

Example 14 Show that the following statement is true.

      p : For any real numbers x, y if $x = y$, then $2x + a = 2y + a$ when $a \in \mathbf{Z}$.

Solution We prove the statement 'p' is true by contrapositive method and by Direct Method.

Direct Method for any real number x, y given

      $x = y$

$\Rightarrow$ $2x = 2y$

$\Rightarrow$ $2x + a = 2y + a$ for some $a \in \mathbf{Z}$.

Contrapositve Method The contrapositive statement of 'p' is "For any real numbers x, y if $2x + a \neq 2y + a$, where $a \in \mathbf{Z}$, then $x \neq y$.

Given $2x + a \neq 2y + a$

$\Rightarrow$ $2x \neq 2y$

$\Rightarrow$ $x \neq y$

Example 15 Check the validity of the statements

(i) r : 100 is a multiple of 4 and 5.

(ii) s : 60 is a multiple of 3 or 5.

Solution (i) Let $p : r \wedge s$

where r : "100 is a multiple of 4" is true

               s : "100 is a multiple of 5" is true

               Hence p is true.

(ii)   Let $q : r \vee s$, where

               r : "60 is a multiple of 3", is true.

               s : "60 is a multiple of 5", is true.

               Hence q is true.

Objective Type Questions

Choose the correct answer out of the four options given against each of the Examples 16 to 18 (M.C.Q.).

Example 16 Which of the following is a statement?

(A) Roses are black.

(B) Mind your own business.

(C) Be punctual.

(D) Do not tell lies.

Solution (A) is the correct answer as the sentences in (B), (C) and (D) are neither true nor false. Infact all these sentences are advices.

Example 17 The negation of the statement

"It is raining and weather is cold." is

(A)   It is not raining and weather is cold.

(B)   It is raining or weather is not cold.

(C)   It is not raining or weather is not cold.

(D)   It is not raining and weather is not cold.

Solution (C) is the correct answer as it satisfies (D_7). The options (A), (B) and (D) do not satisfy (D_7).

Example 18 Which of the following is the converse of the statement?

"If Billu secure good marks, then he will get a bicycle."

(A)   If Billu will not get bicycle, then he will not secure good marks.

(B)   If Billu will get a bicycle, then he will secure good marks.

(C)   If Billu will get a bicycle, then he will not secure good marks.

(D)   If Billu will not get a bicycle, then he will secure good marks.

Solution (B) is the correct answer since the statement $q \rightarrow p$ is the converse of the statement $p \rightarrow q$.

14.3 EXERCISE

Short Answer Type

1.   Which of the following sentences are statements ? Justify

     (i)   A triangle has three sides.

     (ii)   0 is a complex number.

     (iii)   Sky is red.

     (iv)   Every set is an infinite set.

     (v) $15 + 8 > 23$.

     (vi) $y + 9 = 7$.

     (vii)   Where is your bag?

     (viii)   Every square is a rectangle.

     (ix)   Sum of opposite angles of a cyclic quadrilateral is 180°.

     (x) $\sin^2 x + \cos^2 x = 0$

2.   Find the component statements of the following compound statements.

   (i)   Number 7 is prime and odd.

   (ii)   Chennai is in India and is the capital of Tamil Nadu.

   (iii)   The number 100 is divisible by 3, 11 and 5.

   (iv)   Chandigarh is the capital of Haryana and U.P.

   (v) $\sqrt{7}$ is a rational number or an irrational number.

   (vi)   0 is less than every positive integer and every negative integer.

   (vii)   Plants use sunlight, water and carbon dioxide for photosynthesis.

   (viii)   Two lines in a plane either intersect at one point or they are parallel.

   (ix)   A rectangle is a quadrilateral or a 5 - sided polygon.

3.   Write the component statements of the following compound statements and check whether the compound statement is true or false.

   (i)   57 is divisible by 2 or 3.

   (ii)   24 is a multiple of 4 and 6.

   (iii)   All living things have two eyes and two legs.

   (iv)   2 is an even number and a prime number.

4.   Write the negation of the following simple statements

   (i)   The number 17 is prime.

   (ii) $2 + 7 = 6$.

   (iii)   Violets are blue.

   (iv) $\sqrt{5}$ is a rational number.

   (v)   2 is not a prime number.

   (vi)   Every real number is an irrational number.

   (vii)   Cow has four legs.

   (viii)   A leap year has 366 days.

   (ix)   All similar triangles are congruent.

   (x)   Area of a circle is same as the perimeter of the circle.

5.   Translate the following statements into symbolic form

   (i)   Rahul passed in Hindi and English.

   (ii) x and y are even integers.

   (iii)   2, 3 and 6 are factors of 12.

      (iv)   Either x or $x + 1$ is an odd integer.

      (v)   A number is either divisible by 2 or 3.

      (vi)   Either $x = 2$ or $x = 3$ is a root of $3x^2 - x - 10 = 0$

      (vii)   Students can take Hindi or English as an optional paper.

6.   Write down the negation of following compound statements

      (i)   All rational numbers are real and complex.

      (ii)   All real numbers are rationals or irrationals.

      (iii) $x = 2$ and $x = 3$ are roots of the Quadratic equation $x^2 - 5x + 6 = 0$.

      (iv)   A triangle has either 3-sides or 4-sides.

      (v)   35 is a prime number or a composite number.

      (vi)   All prime integers are either even or odd.

      (vii) $|x|$ is equal to either x or $-x$.

      (viii)   6 is divisible by 2 and 3.

7.   Rewrite each of the following statements in the form of conditional statements

      (i)   The square of an odd number is odd.

      (ii)   You will get a sweet dish after the dinner.

      (iii)   You will fail, if you will not study.

      (iv)   The unit digit of an integer is 0 or 5 if it is divisible by 5.

      (v)   The square of a prime number is not prime.

      (vi) $2b = a + c$, if a, b and c are in A.P.

8.   Form the biconditional statement $p \leftrightarrow q$, where

      (i) p : The unit digit of an integer is zero.

               q : It is divisible by 5.

      (ii) p : A natural number n is odd.

               q : Natural number n is not divisible by 2.

      (iii) p : A triangle is an equilateral triangle.

               q : All three sides of a triangle are equal.

9.   Write down the contrapositive of the following statements:

      (i)   If $x = y$ and $y = 3$, then $x = 3$.

(ii)   If n is a natural number, then n is an integer.

(iii)   If all three sides of a triangle are equal, then the triangle is equilateral.

(iv)   If x and y are negative integers, then xy is positive.

(v)   If natural number n is divisible by 6, then n is divisible by 2 and 3.

(vi)   If it snows, then the weather will be cold.

(vii)   If x is a real number such that $0 < x < 1$, then $x^2 < 1$.

10.   Write down the converse of following statements :

(i)   If a rectangle 'R' is a square, then R is a rhombus.

(ii)   If today is Monday, then tomorrow is Tuesday.

(iii)   If you go to Agra, then you must visit Taj Mahal.

(iv)   If the sum of squares of two sides of a triangle is equal to the square of third side of a triangle, then the triangle is right angled.

(v)   If all three angles of a triangle are equal, then the triangle is equilateral.

(vi)   If $x : y = 3 : 2$, then $2x = 3y$.

(vii)   If S is a cyclic quadrilateral, then the opposite angles of S are supplementary.

(viii)   If x is zero, then x is neither positive nor negative.

(ix)   If two triangles are similar, then the ratio of their corresponding sides are equal.

11.   Identify the Quantifiers in the following statements.

(i)   There exists a triangle which is not equilateral.

(ii)   For all real numbers x and y, $xy = yx$.

(iii)   There exists a real number which is not a rational number.

(iv)   For every natural number x, $x + 1$ is also a natural number.

(v)   For all real numbers x with $x > 3$, x^2 is greater than 9.

(vi)   There exists a triangle which is not an isosceles triangle.

(vii)   For all negative integers x, x^3 is also a negative integers.

(viii)   There exists a statement in above statements which is not true.

(ix)   There exists a even prime number other than 2.

(x)   There exists a real number x such that $x^2 + 1 = 0$.

12. Prove by direct method that for any integer '*n*', $n^3 - n$ is always even.

[**Hint:** Two cases (i) *n* is even, (ii) *n* is odd.]

13. Check the validity of the following statement.

(i)   *p* : 125 is divisible by 5 and 7.

(ii)   *q* : 131 is a multiple of 3 or 11.

14. Prove the following statement by contradiction method.

p : The sum of an irrational number and a rational number is irrational.

15. Prove by direct method that for any real numbers *x*, *y* if *x* = *y*, then $x^2 = y^2$.

16. Using contrapositive method prove that if n^2 is an even integer, then *n* is also an even integers.

Objective Type Questions

Choose the correct answer out of the four options given against each of the Exercises 17 to 36 (M.C.Q.).

17. Which of the following is a statement.

(A) *x* is a real number.

(B) Switch off the fan.

(C) 6 is a natural number.

(D) Let me go.

18. Which of the following is not a statement

(A) Smoking is injurious to health.

(B) 2 + 2 = 4

(C) 2 is the only even prime number.

(D) Come here.

19. The connective in the statement

"2 + 7 > 9 or 2 + 7 < 9" is

(A) and

(B) or

(C) >

(D) <

20. The connective in the statement

"Earth revolves round the Sun and Moon is a satellite of earth" is

(A) or

(B) Earth

(C) Sun

(D) and

21. The negation of the statement

"A circle is an ellipse" is

(A) An ellipse is a circle.

(B) An ellipse is not a circle.

(C) A circle is not an ellipse.

(D) A circle is an ellipse.

22. The negation of the statement

"7 is greater than 8" is

(A) 7 is equal to 8.

(B) 7 is not greater than 8.

(C) 8 is less than 7.

(D) none of these

23. The negation of the statement

"72 is divisible by 2 and 3" is

(A)   72 is not divisible by 2 or 72 is not divisible by 3.

(B)   72 is not divisible by 2 and 72 is not divisible by 3.

(C)   72 is divisible by 2 and 72 is not divisible by 3.

(D)   72 is not divisible by 2 and 72 is divisible by 3.

24. The negation of the statement

"Plants take in CO_2 and give out O_2" is

(A)   Plants do not take in CO_2 and do not give out O_2.

(B)   Plants do not take in CO_2 or do not give out O_2.

(C)   Plants take in CO_2 and do not give out O_2.

(D)   Plants take in CO_2 or do not give out O_2.

25. The negation of the statement

"Rajesh or Rajni lived in Bangalore" is

(A)   Rajesh did not live in Bangalore or Rajni lives in Bangalore.

(B)   Rajesh lives in Bangalore and Rajni did not live in Bangalore.

(C)   Rajesh did not live in Bangalore and Rajni did not live in Bangalore.

(D)   Rajesh did not live in Bangalore or Rajni did not live in Bangalore.

26. The negation of the statement

"101 is not a multiple of 3" is

(A)   101 is a multiple of 3.

(B)   101 is a multiple of 2.

(C)   101 is an odd number.

(D)   101 is an even number.

27. The contrapositive of the statement

"If 7 is greater than 5, then 8 is greater than 6" is

(A)   If 8 is greater than 6, then 7 is greater than 5.

(B)   If 8 is not greater than 6, then 7 is greater than 5.

(C)   If 8 is not greater than 6, then 7 is not greater than 5.

(D)   If 8 is greater than 6, then 7 is not greater than 5.

28. The converse of the statement

"If $x > y$, then $x + a > y + a$" is

(A)   If $x < y$, then $x + a < y + a$.

(B)   If $x + a > y + a$, then $x > y$.

(C)   If $x < y$, then $x + a > y + a$.

(D)   If $x > y$, then $x + a < y + a$.

29. The converse of the statement

"If sun is not shining, then sky is filled with clouds" is

(A)   If sky is filled with clouds, then the sun is not shining.

(B)   If sun is shining, then sky is filled with clouds.

(C)   If sky is clear, then sun is shining.

(D)   If sun is not shining, then sky is not filled with clouds.

30.   The contrapositive of the statement

"If p, then q", is

(A)   If q, then p.

(B)   If p, then $\sim q$.

(C)   If $\sim q$, then $\sim p$.

(D)   If $\sim p$, then $\sim q$.

31.   The statement

"If x^2 is not even, then x is not even" is converse of the statement

(A)   If x^2 is odd, then x is even.

(B)   If x is not even, then x^2 is not even.

(C)   If x is even, then x^2 is even.

(D)   If x is odd, then x^2 is even.

32.   The contrapositive of statement

'If Chandigarh is capital of Punjab, then Chandigarh is in India' is

(A)   If Chandigarh is not in India, then Chandigarh is not the capital of Punjab.

(B)   If Chandigarh is in India, then Chandigarh is Capital of Punjab.

(C)   If Chandigarh is not capital of Punjab, then Chandigarh is not capital of India.

(D)   If Chandigarh is capital of Punjab, then Chandigarh is not in India.

33.   Which of the following is the conditional $p \rightarrow q$?

(A) q is sufficient for p.

(B) p is necessary for q.

(C) p only if q.

(D)   if q, then p.

34.   The negation of the statement "The product of 3 and 4 is 9" is

(A)   It is false that the product of 3 and 4 is 9.

(B)   The product of 3 and 4 is 12.

(C)   The product of 3 and 4 is not 12.

(D)   It is false that the product of 3 and 4 is not 9.

35. Which of the following is not a negation of

  "A natural number is greater than zero"

  (A)   A natural number is not greater than zero.

  (B)   It is false that a natural number is greater than zero.

  (C)   It is false that a natural number is not greater than zero.

  (D)   None of the above

36. Which of the following statement is a conjunction ?

  (A)   Ram and Shyam are friends.

  (B)   Both Ram and Shyam are tall.

  (C)   Both Ram and Shyam are enemies.

  (D)   None of the above.

37. State whether the following sentences are statements are not :

  (i)   The angles opposite to equal sides of a triangle are equal.

  (ii)   The moon is a satellite of earth.

  (iii)   May God bless you!

  (iv)   Asia is a continent.

  (v)   How are you?