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Chapter 1

# RELATIONS AND FUNCTIONS

## 1.1 Overview

### 1.1.1 *Relation*

A relation R from a non-empty set A to a non empty set B is a subset of the Cartesian product A × B. The set of all first elements of the ordered pairs in a relation R from a set A to a set B is called the domain of the relation R. The set of all second elements in a relation R from a set A to a set B is called the range of the relation R. The whole set B is called the codomain of the relation R. Note that range is always a subset of codomain.

### 1.1.2 *Types of Relations*

A relation R in a set A is subset of A × A. Thus empty set $\phi$ and A × A are two extreme relations.

(i) A relation R in a set A is called empty relation, if no element of A is related to any element of A, i.e., $R = \phi \subset A \times A$.

(ii) A relation R in a set A is called universal relation, if each element of A is related to every element of A, i.e., $R = A \times A$.

(iii) A relation R in A is said to be reflexive if $a R a$ for all $a \in A$, R is symmetric if $a R b \Rightarrow b R a$, $\forall\ a, b \in A$ and it is said to be transitive if $a R b$ and $b R c \Rightarrow a R c$ $\forall\ a, b, c \in A$. Any relation which is reflexive, symmetric and transitive is called an equivalence relation.

**Note:** An important property of an equivalence relation is that it divides the set into pairwise disjoint subsets called equivalent classes whose collection is called a partition of the set. Note that the union of all equivalence classes gives the whole set.

### 1.1.3 *Types of Functions*

(i) A function $f : X \to Y$ is defined to be one-one (or injective), if the images of distinct elements of X under $f$ are distinct, i.e.,

$$x_1, x_2 \in X,\ f(x_1) = f(x_2) \Rightarrow x_1 = x_2.$$

(ii) A function $f : X \to Y$ is said to be onto (or surjective), if every element of Y is the image of some element of X under $f$, i.e., for every $y \in Y$ there exists an element $x \in X$ such that $f(x) = y$.

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(iii) A function $f: X \to Y$ is said to be one-one and onto (or bijective), if $f$ is both one-one and onto.

### 1.1.4 *Composition of Functions*

(i) Let $f : A \to B$ and $g : B \to C$ be two functions. Then, the composition of $f$ and $g$, denoted by $g \circ f$, is defined as the function $g \circ f : A \to C$ given by

$$g \circ f(x) = g(f(x)), \ \forall \ x \in A.$$

(ii) If $f : A \to B$ and $g : B \to C$ are one-one, then $g \circ f : A \to C$ is also one-one

(iii) If $f : A \to B$ and $g : B \to C$ are onto, then $g \circ f : A \to C$ is also onto.

However, converse of above stated results (ii) and (iii) need not be true. Moreover, we have the following results in this direction.

(iv) Let $f : A \to B$ and $g : B \to C$ be the given functions such that $g \circ f$ is one-one. Then $f$ is one-one.

(v) Let $f : A \to B$ and $g : B \to C$ be the given functions such that $g \circ f$ is onto. Then $g$ is onto.

### 1.1.5 *Invertible Function*

(i) A function $f : X \to Y$ is defined to be invertible, if there exists a function $g : Y \to X$ such that $g \circ f = I_X$ and $f \circ g = I_Y$. The function $g$ is called the inverse of $f$ and is denoted by $f^{-1}$.

(ii) A function $f : X \to Y$ is invertible if and only if $f$ is a bijective function.

(iii) If $f : X \to Y$, $g : Y \to Z$ and $h : Z \to S$ are functions, then $h \circ (g \circ f) = (h \circ g) \circ f$.

(iv) Let $f : X \to Y$ and $g : Y \to Z$ be two invertible functions. Then $g \circ f$ is also invertible with $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

### 1.1.6 *Binary Operations*

(i) A binary operation $*$ on a set A is a function $* : A \times A \to A$. We denote $*(a, b)$ by $a * b$.

(ii) A binary operation $*$ on the set X is called commutative, if $a * b = b * a$ for every $a, b \in X$.

(iii) A binary operation $* : A \times A \to A$ is said to be associative if $(a * b) * c = a * (b * c)$, for every $a, b, c \in A$.

(iv) Given a binary operation $* : A \times A \to A$, an element $e \in A$, if it exists, is called identity for the operation $*$, if $a * e = a = e * a$, $\forall \ a \in A$.

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(v) Given a binary operation $* : A \times A \to A$, with the identity element $e$ in A, an element $a \in A$, is said to be invertible with respect to the operation $*$, if there exists an element $b$ in A such that $a * b = e = b * a$ and $b$ is called the inverse of $a$ and is denoted by $a^{-1}$.

## 1.2 Solved Examples

### Short Answer (S.A.)

**Example 1** Let A = {0, 1, 2, 3} and define a relation R on A as follows:

R = {(0, 0), (0, 1), (0, 3), (1, 0), (1, 1), (2, 2), (3, 0), (3, 3)}.

Is R reflexive? symmetric? transitive?

**Solution** R is reflexive and symmetric, but not transitive since for $(1, 0) \in R$ and $(0, 3) \in R$ whereas $(1, 3) \notin R$.

**Example 2** For the set A = {1, 2, 3}, define a relation R in the set A as follows:

R = {(1, 1), (2, 2), (3, 3), (1, 3)}.

Write the ordered pairs to be added to R to make it the smallest equivalence relation.

**Solution** (3, 1) is the single ordered pair which needs to be added to R to make it the smallest equivalence relation.

**Example 3** Let R be the equivalence relation in the set **Z** of integers given by R = {($a$, $b$) : 2 divides $a - b$}. Write the equivalence class [0].

**Solution** [0] = {0, ± 2, ± 4, ± 6,...}

**Example 4** Let the function $f : \mathbf{R} \to \mathbf{R}$ be defined by $f(x) = 4x - 1$, $\forall\ x \in \mathbf{R}$. Then, show that $f$ is one-one.

**Solution** For any two elements $x_1, x_2 \in \mathbf{R}$ such that $f(x_1) = f(x_2)$, we have

$$4x_1 - 1 = 4x_2 - 1$$

$$\Rightarrow \quad 4x_1 = 4x_2, \text{ i.e., } x_1 = x_2$$

Hence $f$ is one-one.

**Example 5** If $f$ = {(5, 2), (6, 3)}, $g$ = {(2, 5), (3, 6)}, write $f \circ g$.

**Solution** $f \circ g$ = {(2, 2), (3, 3)}

**Example 6** Let $f : \mathbf{R} \to \mathbf{R}$ be the function defined by $f(x) = 4x - 3$ $\forall\ x \in \mathbf{R}$. Then write $f^{-1}$.

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**Solution** Given that $f(x) = 4x - 3 = y$ (say), then

$$4x = y + 3$$

$$\Rightarrow \qquad x = \frac{y+3}{4}$$

Hence $\qquad f^{-1}(y) = \frac{y+3}{4} \qquad \Rightarrow \quad f^{-1}(x) = \frac{x+3}{4}$

**Example 7** Is the binary operation $*$ defined on **Z** (set of integer) by $m * n = m - n + mn \ \forall\, m, n \in \mathbf{Z}$ commutative?

**Solution** No. Since for $1, 2 \in \mathbf{Z}$, $1 * 2 = 1 - 2 + 1.2 = 1$ while $2 * 1 = 2 - 1 + 2.1 = 3$ so that $1 * 2 \neq 2 * 1$.

**Example 8** If $f = \{(5, 2), (6, 3)\}$ and $g = \{(2, 5), (3, 6)\}$, write the range of $f$ and $g$.

**Solution** The range of $f = \{2, 3\}$ and the range of $g = \{5, 6\}$.

**Example 9** If $A = \{1, 2, 3\}$ and $f, g$ are relations corresponding to the subset of $A \times A$ indicated against them, which of $f, g$ is a function? Why?

$$f = \{(1, 3), (2, 3), (3, 2)\}$$

$$g = \{(1, 2), (1, 3), (3, 1)\}$$

**Solution** $f$ is a function since each element of A in the first place in the ordered pairs is related to only one element of A in the second place while $g$ is not a function because 1 is related to more than one element of A, namely, 2 and 3.

**Example 10** If $A = \{a, b, c, d\}$ and $f = \{(a, b), (b, d), (c, a), (d, c)\}$, show that $f$ is one-one from A onto A. Find $f^{-1}$.

**Solution** $f$ is one-one since each element of A is assigned to distinct element of the set A. Also, $f$ is onto since $f(A) = A$. Moreover, $f^{-1} = \{(b, a), (d, b), (a, c), (c, d)\}$.

**Example 11** In the set **N** of natural numbers, define the binary operation $*$ by $m * n = g.c.d\,(m, n)$, $m, n \in \mathbf{N}$. Is the operation $*$ commutative and associative?

**Solution** The operation is clearly commutative since

$$m * n = g.c.d\,(m, n) = g.c.d\,(n, m) = n * m \quad \forall\, m, n \in \mathbf{N}.$$

It is also associative because for $l, m, n \in \mathbf{N}$, we have

$$\begin{aligned} l * (m * n) &= g.\,c.\,d\,(l, g.c.d\,(m, n)) \\ &= g.c.d.\,(g.\,c.\,d\,(l, m), n) \\ &= (l * m) * n. \end{aligned}$$

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## Long Answer (L.A.)

**Example 12** In the set of natural numbers **N**, define a relation R as follows: $\forall\, n, m \in \mathbf{N}$, $n$R$m$ if on division by 5 each of the integers $n$ and $m$ leaves the remainder less than 5, i.e. one of the numbers 0, 1, 2, 3 and 4. Show that R is equivalence relation. Also, obtain the pairwise disjoint subsets determined by R.

**Solution** R is reflexive since for each $a \in \mathbf{N}$, $a$R$a$. R is symmetric since if $a$R$b$, then $b$R$a$ for $a, b \in \mathbf{N}$. Also, R is transitive since for $a, b, c \in \mathbf{N}$, if $a$R$b$ and $b$R$c$, then $a$R$c$. Hence R is an equivalence relation in **N** which will partition the set **N** into the pairwise disjoint subsets. The equivalent classes are as mentioned below:

$A_0 = \{5, 10, 15, 20 ...\}$

$A_1 = \{1, 6, 11, 16, 21 ...\}$

$A_2 = \{2, 7, 12, 17, 22, ...\}$

$A_3 = \{3, 8, 13, 18, 23, ...\}$

$A_4 = \{4, 9, 14, 19, 24, ...\}$

It is evident that the above five sets are pairwise disjoint and

$$A_0 \cup A_1 \cup A_2 \cup A_3 \cup A_4 = \bigcup_{i=0}^{4} A_i = \mathbf{N}.$$

**Example 13** Show that the function $f : \mathbf{R} \to \mathbf{R}$ defined by $f(x) = \dfrac{x}{x^2+1}, \forall\, x \in \mathbf{R}$, is neither one-one nor onto.

**Solution** For $x_1, x_2 \in \mathbf{R}$, consider

$$f(x_1) = f(x_2)$$

$$\Rightarrow \frac{x_1}{x_1^2+1} = \frac{x_2}{x_2^2+1}$$

$$\Rightarrow x_1 x_2^2 + x_1 = x_2 x_1^2 + x_2$$

$$\Rightarrow x_1 x_2 (x_2 - x_1) = x_2 - x_1$$

$$\Rightarrow x_1 = x_2 \text{ or } x_1 x_2 = 1$$

We note that there are point, $x_1$ and $x_2$ with $x_1 \neq x_2$ and $f(x_1) = f(x_2)$, for instance, if we take $x_1 = 2$ and $x_2 = \frac{1}{2}$, then we have $f(x_1) = \frac{2}{5}$ and $f(x_2) = \frac{2}{5}$ but $2 \neq \frac{1}{2}$. Hence $f$ is not one-one. Also, $f$ is not onto for if so then for $1 \in \mathbf{R}\ \exists\ x \in \mathbf{R}$ such that $f(x) = 1$

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which gives $\frac{x}{x^2+1}=1$. But there is no such $x$ in the domain **R**, since the equation $x^2 - x + 1 = 0$ does not give any real value of $x$.

**Example 14** Let $f, g : \mathbf{R} \to \mathbf{R}$ be two functions defined as $f(x) = |x| + x$ and $g(x) = |x| - x \ \forall\ x \in \mathbf{R}$. Then, find $f \circ g$ and $g \circ f$.

**Solution** Here $f(x) = |x| + x$ which can be redefined as

$$f(x) = \begin{cases} 2x \text{ if } x \geq 0 \\ 0 \text{ if } x < 0 \end{cases}$$

Similarly, the function $g$ defined by $g(x) = |x| - x$ may be redefined as

$$g(x) = \begin{cases} 0 \text{ if } x \geq 0 \\ -2x \text{ if } x < 0 \end{cases}$$

Therefore, $g \circ f$ gets defined as :

For $x \geq 0$, $(g \circ f)(x) = g(f(x)) = g(2x) = 0$

and for $x < 0$, $(g \circ f)(x) = g(f(x)) = g(0) = 0$.

Consequently, we have $(g \circ f)(x) = 0,\ \forall\ x \in \mathbf{R}$.

Similarly, $f \circ g$ gets defined as:

For $x \geq 0$, $(f \circ g)(x) = f(g(x)) = f(0) = 0$,

and for $x < 0$, $(f \circ g)(x) = f(g(x)) = f(-2x) = -4x$.

i.e. $$(f \circ g)(x) = \begin{cases} 0, x > 0 \\ -4x, x < 0 \end{cases}$$

**Example 15** Let **R** be the set of real numbers and $f : \mathbf{R} \to \mathbf{R}$ be the function defined by $f(x) = 4x + 5$. Show that $f$ is invertible and find $f^{-1}$.

**Solution** Here the function $f : \mathbf{R} \to \mathbf{R}$ is defined as $f(x) = 4x + 5 = y$ (say). Then

$$4x = y - 5 \quad \text{or} \quad x = \frac{y-5}{4}.$$

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This leads to a function $g : \mathbf{R} \to \mathbf{R}$ defined as

$$g(y) = \frac{y-5}{4}.$$

Therefore, $(g \circ f)(x) = g(f(x)) = g(4x + 5)$

$$= \frac{4x+5-5}{4} = x$$

or $g \circ f = I_R$

Similarly $(f \circ g)(y) = f(g(y))$

$$= f\left(\frac{y-5}{4}\right)$$

$$= 4\left(\frac{y-5}{4}\right) + 5 = y$$

or $f \circ g = I_R$.

Hence $f$ is invertible and $f^{-1} = g$ which is given by

$$f^{-1}(x) = \frac{x-5}{4}$$

**Example 16** Let $*$ be a binary operation defined on **Q**. Find which of the following binary operations are associative

(i) $a * b = a - b$ for $a, b \in \mathbf{Q}$.

(ii) $a * b = \frac{ab}{4}$ for $a, b \in \mathbf{Q}$.

(iii) $a * b = a - b + ab$ for $a, b \in \mathbf{Q}$.

(iv) $a * b = ab^2$ for $a, b \in \mathbf{Q}$.

**Solution**

(i) $*$ is not associative for if we take $a = 1$, $b = 2$ and $c = 3$, then

$(a * b) * c = (1 * 2) * 3 = (1 - 2) * 3 = -1 - 3 = -4$ and

$a * (b * c) = 1 * (2 * 3) = 1 * (2 - 3) = 1 - (-1) = 2.$

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Thus $(a * b) * c \neq a * (b * c)$ and hence $*$ is not associative.

(ii) $*$ is associative since **Q** is associative with respect to multiplication.

(iii) $*$ is not associative for if we take $a = 2$, $b = 3$ and $c = 4$, then

$(a * b) * c = (2 * 3) * 4 = (2 – 3 + 6) * 4 = 5 * 4 = 5 – 4 + 20 = 21$, and

$a * (b * c) = 2 * (3 * 4) = 2 * (3 – 4 + 12) = 2 * 11 = 2 – 11 + 22 = 13$

Thus $(a * b) * c \neq a * (b * c)$ and hence $*$ is not associative.

(iv) $*$ is not associative for if we take $a = 1$, $b = 2$ and $c = 3$, then $(a * b) * c = (1 * 2) * 3 = 4 * 3 = 4 \times 9 = 36$ and $a * (b * c) = 1 * (2 * 3) = 1 * 18 = 1 \times 18^2 = 324$.

Thus $(a * b) * c \neq a * (b * c)$ and hence $*$ is not associative.

## Objective Type Questions

Choose the correct answer from the given four options in each of the Examples 17 to 25.

**Example 17** Let R be a relation on the set **N** of natural numbers defined by $n$R$m$ if $n$ divides $m$. Then R is

(A) Reflexive and symmetric
(B) Transitive and symmetric
(C) Equivalence
(D) Reflexive, transitive but not symmetric

**Solution** The correct choice is (D).
Since $n$ divides $n$, $\forall n \in$ **N**, R is reflexive. R is not symmetric since for 3, 6 $\in$ **N**, 3 R 6 $\neq$ 6 R 3. R is transitive since for $n$, $m$, $r$ whenever $n/m$ and $m/r \Rightarrow n/r$, i.e., $n$ divides $m$ and $m$ divides $r$, then $n$ will devide $r$.

**Example 18** Let L denote the set of all straight lines in a plane. Let a relation R be defined by $l$R$m$ if and only if $l$ is perpendicular to $m$ $\forall l, m \in$ L. Then R is

(A) reflexive
(B) symmetric
(C) transitive
(D) none of these

**Solution** The correct choice is (B).

**Example 19** Let **N** be the set of natural numbers and the function $f$ : **N** $\rightarrow$ **N** be defined by $f(n) = 2n + 3$ $\forall n \in$ **N**. Then $f$ is

(A) surjective
(B) injective
(C) bijective
(D) none of these

**Solution** (B) is the correct option.

**Example 20** Set A has 3 elements and the set B has 4 elements. Then the number of

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injective mappings that can be defined from A to B is

(A) 144 (B) 12
(C) 24 (D) 64

**Solution** The correct choice is (C). The total number of injective mappings from the set containing 3 elements into the set containing 4 elements is ${}^4P_3 = 4! = 24$.

**Example 21** Let $f : \mathbf{R} \to \mathbf{R}$ be defined by $f(x) = \sin x$ and $g : \mathbf{R} \to \mathbf{R}$ be defined by $g(x) = x^2$, then $f \circ g$ is

(A) $x^2 \sin x$ (B) $(\sin x)^2$
(C) $\sin x^2$ (D) $\dfrac{\sin x}{x^2}$

**Solution** (C) is the correct choice.

**Example 22** Let $f : \mathbf{R} \to \mathbf{R}$ be defined by $f(x) = 3x - 4$. Then $f^{-1}(x)$ is given by

(A) $\dfrac{x+4}{3}$ (B) $\dfrac{x}{3} - 4$
(C) $3x + 4$ (D) None of these

**Solution** (A) is the correct choice.

**Example 23** Let $f : \mathbf{R} \to \mathbf{R}$ be defined by $f(x) = x^2 + 1$. Then, pre-images of 17 and – 3, respectively, are

(A) $\phi$, {4, – 4} (B) {3, – 3}, $\phi$
(C) {4, –4}, $\phi$ (D) {4, – 4, {2, – 2}

**Solution** (C) is the correct choice since for $f^{-1}(17) = x \Rightarrow f(x) = 17$ or $x^2 + 1 = 17$ $\Rightarrow x = \pm 4$ or $f^{-1}(17) = \{4, -4\}$ and for $f^{-1}(-3) = x \Rightarrow f(x) = -3 \Rightarrow x^2 + 1 = -3 \Rightarrow x^2 = -4$ and hence $f^{-1}(-3) = \phi$.

**Example 24** For real numbers $x$ and $y$, define $x$R$y$ if and only if $x - y + \sqrt{2}$ is an irrational number. Then the relation R is

(A) reflexive (B) symmetric
(C) transitive (D) none of these

**Solution** (A) is the correct choice.

Fill in the blanks in each of the Examples 25 to 30.

**Example 25** Consider the set A = {1, 2, 3} and R be the smallest equivalence relation on A, then R = ________

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**Solution** R = {(1, 1), (2, 2), (3, 3)}.

**Example 26** The domain of the function $f : \mathbf{R} \rightarrow \mathbf{R}$ defined by $f(x) = \sqrt{x^2 - 3x + 2}$ is ________.

**Solution** Here $x^2 - 3x + 2 \geq 0$

$\Rightarrow \quad (x - 1)(x - 2) \geq 0$

$\Rightarrow \quad x \leq 1$ or $x \geq 2$

Hence the domain of $f = (-\infty, 1] \cup [2, \infty)$

**Example 27** Consider the set A containing $n$ elements. Then, the total number of injective functions from A onto itself is ________.

**Solution** $n!$

**Example 28** Let **Z** be the set of integers and R be the relation defined in **Z** such that $a$R$b$ if $a - b$ is divisible by 3. Then R partitions the set **Z** into ________ pairwise disjoint subsets.

**Solution** Three.

**Example 29** Let **R** be the set of real numbers and $*$ be the binary operation defined on **R** as $a * b = a + b - ab \;\; \forall \; a, b \in \mathbf{R}$. Then, the identity element with respect to the binary operation $*$ is _______.

**Solution** 0 is the identity element with respect to the binary operation $*$.

State **True** or **False** for the statements in each of the Examples 30 to 34.

**Example 30** Consider the set A = {1, 2, 3} and the relation R = {(1, 2), (1, 3)}. R is a transitive relation.

**Solution** True.

**Example 31** Let A be a finite set. Then, each injective function from A into itself is not surjective.

**Solution** False.

**Example 32** For sets A, B and C, let $f : A \rightarrow B$, $g : B \rightarrow C$ be functions such that $g \circ f$ is injective. Then both $f$ and $g$ are injective functions.

**Solution** False.

**Example 33** For sets A, B and C, let $f : A \rightarrow B$, $g : B \rightarrow C$ be functions such that $g \circ f$ is surjective. Then $g$ is surjective

**Solution** True.

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**Example 34** Let **N** be the set of natural numbers. Then, the binary operation $*$ in **N** defined as $a * b = a + b$, $\forall$ $a, b \in$ **N** has identity element.

**Solution** False.

## 1.3 EXERCISE

### Short Answer (S.A.)

**1.** Let A = {$a$, $b$, $c$} and the relation R be defined on A as follows:

$$R = \{(a, a), (b, c), (a, b)\}.$$

Then, write minimum number of ordered pairs to be added in **R** to make **R** reflexive and transitive.

**2.** Let D be the domain of the real valued function $f$ defined by $f(x) = \sqrt{25-x^2}$. Then, write D.

**3.** Let $f, g$ : **R** $\rightarrow$ **R** be defined by $f(x) = 2x + 1$ and $g(x) = x^2 - 2$, $\forall x \in$ **R**, respectively. Then, find $g \circ f$.

**4.** Let $f$ : **R** $\rightarrow$ **R** be the function defined by $f(x) = 2x - 3$ $\forall x \in$ R. write $f^{-1}$.

**5.** If A = {$a$, $b$, $c$, $d$} and the function $f = \{(a, b), (b, d), (c, a), (d, c)\}$, write $f^{-1}$.

**6.** If $f$ : **R** $\rightarrow$ **R** is defined by $f(x) = x^2 - 3x + 2$, write $f(f(x))$.

**7.** Is $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ a function? If $g$ is described by $g(x) = \alpha x + \beta$, then what value should be assigned to $\alpha$ and $\beta$.

**8.** Are the following set of ordered pairs functions? If so, examine whether the mapping is injective or surjective.

(i) {($x$, $y$): $x$ is a person, $y$ is the mother of $x$}.

(ii){($a$, $b$): $a$ is a person, $b$ is an ancestor of $a$}.

**9.** If the mappings $f$ and $g$ are given by

$f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(2, 3), (5, 1), (1, 3)\}$, write $f \circ g$.

**10.** Let **C** be the set of complex numbers. Prove that the mapping $f$ : **C** $\rightarrow$ **R** given by $f(z) = |z|$, $\forall$ $z \in$ **C**, is neither one-one nor onto.

**11.** Let the function $f$ : **R** $\rightarrow$ **R** be defined by $f(x) = \cos x$, $\forall$ $x \in$ **R**. Show that $f$ is neither one-one nor onto.

**12.** Let X = {1, 2, 3}and Y = {4, 5}. Find whether the following subsets of X $\times$Y are functions from X to Y or not.

(i) $f = \{(1, 4), (1, 5), (2, 4), (3, 5)\}$ (ii) $g = \{(1, 4), (2, 4), (3, 4)\}$

(iii) $h = \{(1,4), (2, 5), (3, 5)\}$ (iv) $k = \{(1,4), (2, 5)\}$.

**13.** If functions $f$ : A $\rightarrow$ B and $g$ : B $\rightarrow$ A satisfy $g \circ f = I_A$, then show that $f$ is one-one and $g$ is onto.

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**14.** Let $f : \mathbf{R} \to \mathbf{R}$ be the function defined by $f(x) = \dfrac{1}{2-\cos x}$ $\forall\, x \in \mathbf{R}$. Then, find the range of $f$.

**15.** Let $n$ be a fixed positive integer. Define a relation R in $\mathbf{Z}$ as follows: $\forall\, a, b \in \mathbf{Z}$, $a$R$b$ if and only if $a - b$ is divisible by $n$ . Show that R is an equivalance relation.

## Long Answer (L.A.)

**16.** If A = {1, 2, 3, 4 }, define relations on A which have properties of being:

(a) reflexive, transitive but not symmetric

(b) symmetric but neither reflexive nor transitive

(c) reflexive, symmetric and transitive.

**17.** Let R be relation defined on the set of natural number **N** as follows:

R = {($x$, $y$): $x \in \mathbf{N}$, $y \in \mathbf{N}$, $2x + y = 41$}. Find the domain and range of the relation R. Also verify whether R is reflexive, symmetric and transitive.

**18.** Given A = {2, 3, 4}, B = {2, 5, 6, 7}. Construct an example of each of the following:

(a) an injective mapping from A to B

(b) a mapping from A to B which is not injective

(c) a mapping from B to A.

**19.** Give an example of a map

(i) which is one-one but not onto

(ii) which is not one-one but onto

(iii) which is neither one-one nor onto.

**20.** Let A = $\mathbf{R}$ – {3}, B = $\mathbf{R}$ – {1}. Let $f$ : A $\to$ B be defined by $f(x) = \dfrac{x-2}{x-3}$ $\forall\, x \in$ A . Then show that $f$ is bijective.

**21.** Let A = [–1, 1]. Then, discuss whether the following functions defined on A are one-one, onto or bijective:

(i) $f(x) = \dfrac{x}{2}$ (ii) $g(x) = |x|$

(iii) $h(x) = x|x|$ (iv) $k(x) = x^2$.

**22.** Each of the following defines a relation on **N**:

(i) $x$ is greater than $y$, $x, y \in \mathbf{N}$

(ii) $x + y = 10$, $x, y \in \mathbf{N}$

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(iii) $x\, y$ is square of an integer $x, y \in$ **N**

(iv) $x + 4y = 10$ $x, y \in$ **N**.

Determine which of the above relations are reflexive, symmetric and transitive.

**23.** Let A = {1, 2, 3, ... 9} and R be the relation in A ×A defined by $(a, b)$ R $(c, d)$ if $a + d = b + c$ for $(a, b), (c, d)$ in A ×A. Prove that R is an equivalence relation and also obtain the equivalent class [(2, 5)].

**24.** Using the definition, prove that the function $f : A \to B$ is invertible if and only if $f$ is both one-one and onto.

**25.** Functions $f, g : \mathbf{R} \to \mathbf{R}$ are defined, respectively, by $f(x) = x^2 + 3x + 1$, $g(x) = 2x - 3$, find

(i) $f \circ g$ (ii) $g \circ f$ (iii) $f \circ f$ (iv) $g \circ g$

**26.** Let $*$ be the binary operation defined on **Q**. Find which of the following binary operations are commutative

(i) $a * b = a - b \;\forall\; a, b \in \mathbf{Q}$
(ii) $a * b = a^2 + b^2 \;\forall\; a, b \in \mathbf{Q}$
(iii) $a * b = a + ab \;\forall\; a, b \in \mathbf{Q}$
(iv) $a * b = (a - b)^2 \;\forall\; a, b \in \mathbf{Q}$

**27.** Let $*$ be binary operation defined on **R** by $a * b = 1 + ab$, $\forall\; a, b \in \mathbf{R}$. Then the operation $*$ is

(i) commutative but not associative
(ii) associative but not commutative
(iii) neither commutative nor associative
(iv) both commutative and associative

## Objective Type Questions

Choose the correct answer out of the given four options in each of the Exercises from 28 to 47 (M.C.Q.).

**28.** Let T be the set of all triangles in the Euclidean plane, and let a relation R on T be defined as $a$R$b$ if $a$ is congruent to $b$ $\forall\; a, b \in$ T. Then R is

(A) reflexive but not transitive
(B) transitive but not symmetric
(C) equivalence
(D) none of these

**29.** Consider the non-empty set consisting of children in a family and a relation R defined as $a$R$b$ if $a$ is brother of $b$. Then R is

(A) symmetric but not transitive
(B) transitive but not symmetric
(C) neither symmetric nor transitive
(D) both symmetric and transitive

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**30.** The maximum number of equivalence relations on the set A = {1, 2, 3} are

(A) 1 (B) 2

(C) 3 (D) 5

**31.** If a relation R on the set {1, 2, 3} be defined by R = {(1, 2)}, then R is

(A) reflexive (B) transitive

(C) symmetric (D) none of these

**32.** Let us define a relation R in **R** as $a$R$b$ if $a \geq b$. Then R is

(A) an equivalence relation (B) reflexive, transitive but not symmetric

(C) symmetric, transitive but not reflexive (D) neither transitive nor reflexive but symmetric.

**33.** Let A = {1, 2, 3} and consider the relation

R = {1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1,3)}.

Then R is

(A) reflexive but not symmetric (B) reflexive but not transitive

(C) symmetric and transitive (D) neither symmetric, nor transitive

**34.** The identity element for the binary operation $*$ defined on Q ~ {0} as

$$a * b = \frac{ab}{2} \quad \forall\, a, b \in Q \sim \{0\} \text{ is}$$

(A) 1 (B) 0

(C) 2 (D) none of these

**35.** If the set A contains 5 elements and the set B contains 6 elements, then the number of one-one and onto mappings from A to B is

(A) 720 (B) 120

(C) 0 (D) none of these

**36.** Let A = {1, 2, 3, ...$n$} and B = {$a$, $b$}. Then the number of surjections from A into B is

(A) ${}^nP_2$ (B) $2^n - 2$

(C) $2^n - 1$ (D) None of these

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**37.** Let $f : \mathbf{R} \to \mathbf{R}$ be defined by $f(x) = \frac{1}{x}$ $\forall x \in \mathbf{R}$. Then $f$ is

(A) one-one (B) onto

(C) bijective (D) $f$ is not defined

**38.** Let $f : \mathbf{R} \to \mathbf{R}$ be defined by $f(x) = 3x^2 - 5$ and $g : \mathbf{R} \to \mathbf{R}$ by $g(x) = \frac{x}{x^2+1}$.

Then $g \circ f$ is

(A) $\frac{3x^2-5}{9x^4-30x^2+26}$ (B) $\frac{3x^2-5}{9x^4-6x^2+26}$

(C) $\frac{3x^2}{x^4+2x^2-4}$ (D) $\frac{3x^2}{9x^4+30x^2-2}$

**39.** Which of the following functions from **Z** into **Z** are bijections?

(A) $f(x) = x^3$ (B) $f(x) = x + 2$

(C) $f(x) = 2x + 1$ (D) $f(x) = x^2 + 1$

**40.** Let $f : \mathbf{R} \to \mathbf{R}$ be the functions defined by $f(x) = x^3 + 5$. Then $f^{-1}(x)$ is

(A) $(x+5)^{\frac{1}{3}}$ (B) $(x-5)^{\frac{1}{3}}$

(C) $(5-x)^{\frac{1}{3}}$ (D) $5 - x$

**41.** Let $f : A \to B$ and $g : B \to C$ be the bijective functions. Then $(g \circ f)^{-1}$ is

(A) $f^{-1} \circ g^{-1}$ (B) $f \circ g$

(C) $g^{-1} \circ f^{-1}$ (D) $g \circ f$

**42.** Let $f : \mathbf{R} - \left\{\frac{3}{5}\right\} \to \mathbf{R}$ be defined by $f(x) = \frac{3x+2}{5x-3}$. Then

(A) $f^{-1}(x) = f(x)$ (B) $f^{-1}(x) = -f(x)$

(C) $(f \circ f)\, x = -x$ (D) $f^{-1}(x) = \frac{1}{19} f(x)$

**43.** Let $f : [0, 1] \to [0, 1]$ be defined by $f(x) = \begin{cases} x, & \text{if } x \text{ is rational} \\ 1-x, & \text{if } x \text{ is irrational} \end{cases}$

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Then $(f\,o\,f)\,x$ is

(A) constant (B) $1 + x$

(C) $x$ (D) none of these

44. Let $f : [2, \infty) \to \mathbf{R}$ be the function defined by $f(x) = x^2 - 4x + 5$, then the range of $f$ is

(A) $\mathbf{R}$ (B) $[1, \infty)$

(C) $[4, \infty)$ (B) $[5, \infty)$

45. Let $f : \mathbf{N} \to \mathbf{R}$ be the function defined by $f(x) = \frac{2x-1}{2}$ and $g : \mathbf{Q} \to \mathbf{R}$ be another function defined by $g(x) = x + 2$. Then $(g\,o\,f)\,\frac{3}{2}$ is

(A) 1 (B) 1

(C) $\frac{7}{2}$ (B) none of these

46. Let $f : \mathbf{R} \to \mathbf{R}$ be defined by

$$f(x) = \begin{cases} 2x : x > 3 \\ x^2 : 1 < x \le 3 \\ 3x : x \le 1 \end{cases}$$

Then $f(-1) + f(2) + f(4)$ is

(A) 9 (B) 14

(C) 5 (D) none of these

47. Let $f : \mathbf{R} \to \mathbf{R}$ be given by $f(x) = \tan x$. Then $f^{-1}(1)$ is

(A) $\frac{\pi}{4}$ (B) $\{n\pi + \frac{\pi}{4} : n \in Z\}$

(C) does not exist (D) none of these

Fill in the blanks in each of the Exercises 48 to 52.

48. Let the relation R be defined in $\mathbf{N}$ by $a$R$b$ if $2a + 3b = 30$. Then R = ______.

49. Let the relation R be defined on the set A = {1, 2, 3, 4, 5} by R = $\{(a, b) : |a^2 - b^2| < 8\}$. Then R is given by ______.

50. Let $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(2, 3), (5, 1), (1, 3)\}$. Then $g\,o\,f$ = ______ and $f\,o\,g$ = ______.

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**51.** Let $f : \mathbf{R} \rightarrow \mathbf{R}$ be defined by $f(x) = \frac{x}{\sqrt{1+x^2}}$. Then $(f \circ f \circ f)(x)$ = ________

**52.** If $f(x) = (4 - (x-7)^3\}$, then $f^{-1}(x)$ = ________.

State **True** or **False** for the statements in each of the Exercises 53 to 63.

**53.** Let R = {(3, 1), (1, 3), (3, 3)} be a relation defined on the set A = {1, 2, 3}. Then R is symmetric, transitive but not reflexive.

**54.** Let $f : \mathbf{R} \rightarrow \mathbf{R}$ be the function defined by $f(x) = \sin(3x+2)$ $\forall x \in$ R. Then $f$ is invertible.

**55.** Every relation which is symmetric and transitive is also reflexive.

**56.** An integer $m$ is said to be related to another integer $n$ if $m$ is a integral multiple of $n$. This relation in **Z** is reflexive, symmetric and transitive.

**57.** Let A = {0, 1} and **N** be the set of natural numbers. Then the mapping $f : \mathbf{N} \rightarrow A$ defined by $f(2n-1) = 0$, $f(2n) = 1$, $\forall n \in \mathbf{N}$, is onto.

**58.** The relation R on the set A = {1, 2, 3} defined as R = {{1, 1), (1, 2), (2, 1), (3, 3)} is reflexive, symmetric and transitive.

**59.** The composition of functions is commutative.

**60.** The composition of functions is associative.

**61.** Every function is invertible.

**62.** A binary operation on a set has always the identity element.

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