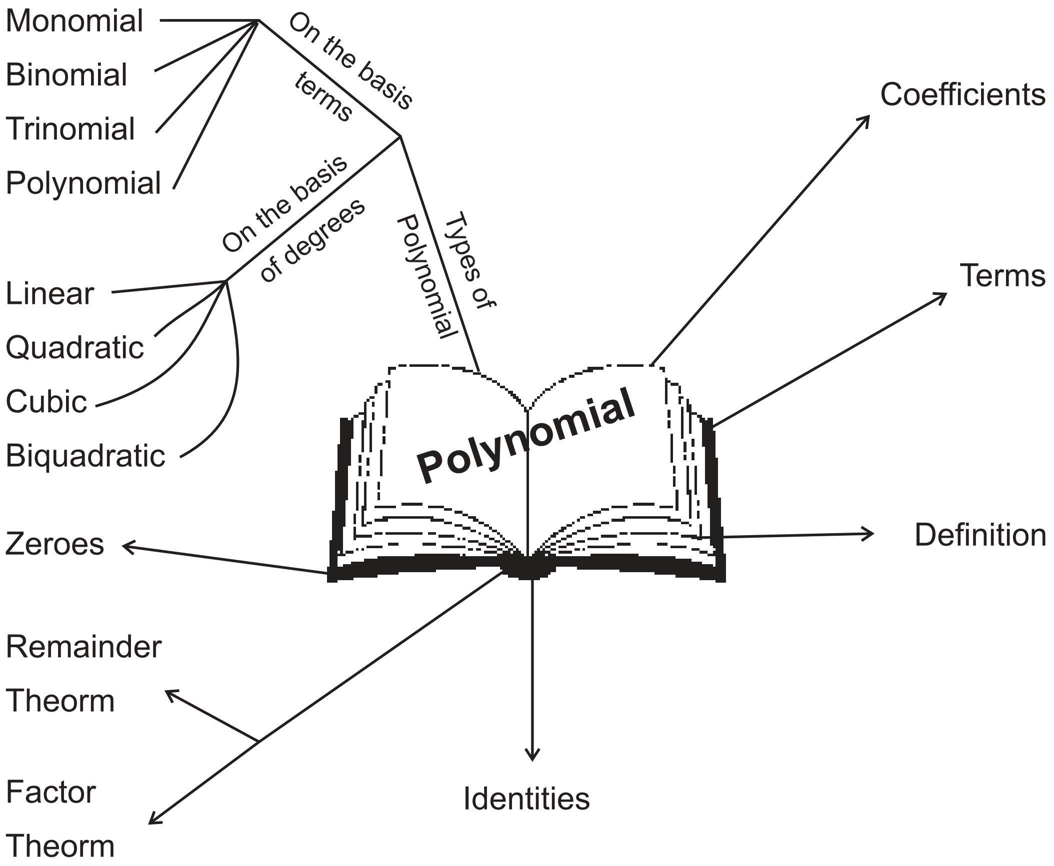
CHAPTER-2
POLYNOMIALS
MIND MAP



CHAPTER-2

POLYNOMIALS

KEY POINTS

1. A Polynomial p(x) in one variable x is an algebraic expression in x of the form $p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_2x^2 + a_1x + a_0$, where

   (i) $a_0, a_1, a_2 ..... a_n$ are constants and $a_n \neq 0$

   (ii) $a_0, a_1, a_2..... a_n$ are respectively the coefficients of $x^0, x^1, x^2, ..., x^n$

   (iii) Each of $a_nx^n, a_{n-1}x^{n-1}, a_{n-2}x^{n-2}, ..., a_2x^2, a_1x$, a are called terms of the polynomial.

   (iv) n is called the degree of the polynomial where n is a non-negative integers.

2. **Degree of the Polynomial :** Highest power of x in the algebraic expression is called the degree of the polynomial.

3. **Different types of polynomials :**
   Generally, we divide the polynomials in the following categories :

   (i) Based on degrees

	Degree	Polynomial	General form	Examples
(a)	1	Linear	$ax + b$,	$x + 1$, $2x$ etc.
(b)	2	Quadratic	$ax^2 + bx + c$,	$4x^2 + 5x + \frac{2}{3}$ etc.
(c)	3	Cubic	$ax^3 + bx^2 + cx + d$,	$x^3 - 3x^2 + 5$ etc.
(d)	4	Biquadratic	$ax^4 + bx^3 + cx^2 + dx + e$,	$x^4 - 16$ etc.

   a, b, c, d, e are real constants and $a \neq 0$.

   Note : A polynomial of degree five or more than five does not have any particular name. Such a polynomial is usually called a polynomial of degree five or six or ... etc.

   (ii) Based on Number of Terms:

	No. of Terms	Polynominal	Examples
(a)	1	Monomial	5, $3x$, $\frac{1}{3}y$ etc.
(b)	2	Binomial	$\sqrt{3} + 6x$, $x - 5y$, $x^2 + 2$ etc.
(c)	3	Trinomial	$\sqrt{2}x^2 + 4x + 2$, $5y^4 + 2y + 6$ etc.

Note : A polynomial having four or more than four terms does not have particular name. These are simply called polynomials.

(iii) Zero degree polynomial or non-zero constant polynomial.

Any non-zero number (constant) is regarded as polynomial of degree zero or zero degree polynomial. i.e., $p(x) = a$ where $a \neq 0$ is a zero degree polynomial, since we can write $p(x) = a$,

as $\quad p(x) = ax^0$

e.g., $\quad 5 = 5x^0 \quad , \quad \frac{\sqrt{7}}{2} = \frac{\sqrt{7}}{2}x^0$

(iv) Zero Polynomial : A polynomial whose all coefficients are zero is called as zero polynomial i.e., $p(x) = 0$. The degree of zero polynomial is not defined or we can not determine the degree of zero polynomial.

4. For a polynomial $p(x)$ if $p(a) = 0$ where a is a real number we say that 'a' is a zero of the polynomial.

5. If $p(x)$ is any polynomial of degree greater than or equal to 1 and $p(x)$ is divided by a linear polynomial $x - a$, then the remainder is $p(a)$. This is called remainder theorem.

6. If $p(x)$ is a polynomial of degree ≥ 1 and 'a' is any real number then

(i) $(x - a)$ is a factor of $p(x)$, if $p(a) = 0$ and

(ii) $p(a) = 0$ if $(x - a)$ is a factor of $p(x)$.

This is called factor theorem.

7. A polynomial of degree 'n' can have at most n zeroes.

• Some algebraic identities :–

(i) $(x+y)^2 = x^2 + 2xy + y^2$

(ii) $(x-y)^2 = x^2 - 2xy + y^2$

(iii) $x^2 - y^2 = (x+y)(x-y)$

(iv) $(x+a)(x+b) = x^2 + (a+b)x + ab$

(v) $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

(vi) $(x+y)^3 = x^3 + y^3 + 3xy\,(x+y) = x^3 + y^3 + 3x^2y + 3xy^2$

(vii) $(x-y)^3 = x^3 - y^3 - 3xy\,(x-y) = x^3 - y^3 - 3x^2y + 3xy^2$

(viii) $x^3 + y^3 = (x+y)\,(x^2 - xy + y^2)$

ix) $x^3 - y^3 = (x-y)\,(x^2 + xy + y^2)$

x) $x^3 + y^3 + z^3 - 3xyz = (x+y+z)\,(x^2 + y^2 + z^2 - xy - yz - zx)$

$$= \frac{1}{2}\,(x+y+z)\,\{(x-y)^2 + (y-z)^2 + (z-x)^2\}$$

xi) If $x+y+z = 0$, then $x^3 + y^3 + z^3 = 3xyz$