Observe that as the speed increases, time taken to cover the same distance decreases.

As Zaheeda doubles her speed by running, time reduces to half. As she increases her speed to three times by cycling, time decreases to one third. Similarly, as she increases her speed to 15 times, time decreases to one fifteenth. (Or, in other words the ratio by which time decreases is inverse of the ratio by which the corresponding speed increases). Can we say that speed and time change inversely in proportion?

> Multiplicative inverse of a number is its reciprocal. Thus, $\frac{1}{2}$ is the inverse of 2 and vice versa. (Note that $2 \times \frac{1}{2} = \frac{1}{2} \times 2 = 1$).

Let us consider another example. A school wants to spend Rs 6000 on mathematics textbooks. How many books could be bought at Rs 40 each? Clearly 150 books can be bought. If the price of a textbook is more than Rs 40, then the number of books which could be purchased with the same amount of money would be less than 150. Observe the following table.

Price of each book (in Rs)	40	50	60	75	80	100
Number of books that can be bought	150	120	100	80	75	60

What do you observe? You will appreciate that as the price of the books increases, the number of books that can be bought, keeping the fund constant, will decrease.

Ratio by which the price of books increases when going from 40 to 50 is 4 : 5, and the ratio by which the corresponding number of books decreases from 150 to 120 is 5 : 4. This means that the two ratios are inverses of each other.

Notice that the product of the corresponding values of the two quantities is constant; that is, $40 \times 150 = 50 \times 120 = 6000$.

If we represent the price of one book as x and the number of books bought as y, then as x increases y decreases and vice-versa. It is important to note that the product xy remains constant. We say that x varies inversely with y and y varies inversely with x. *Thus two quantities x and y are said to vary in inverse proportion, if there exists a relation of the type $xy = k$ between them, k being a constant. If y_1, y_2 are the values of y corresponding to the values x_1, x_2 of x respectively then $x_1y_1 = x_2y_2$ (= k), or* $\frac{x_1}{x_2} = \frac{y_2}{y_1}$.

We say that x and y are in **inverse proportion**.

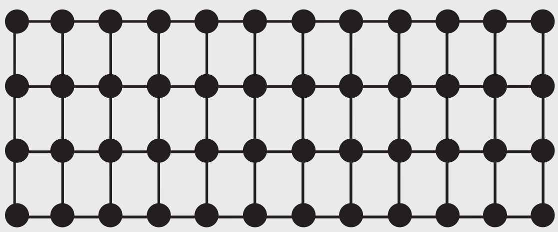
Hence, in this example, cost of a book and number of books purchased in a fixed amount are inversely proportional. Similarly, speed of a vehicle and the time taken to cover a fixed distance changes in inverse proportion.

Think of more such examples of pairs of quantities that vary in inverse proportion. You may now have a look at the furniture – arranging problem, stated in the introductory part of this chapter.

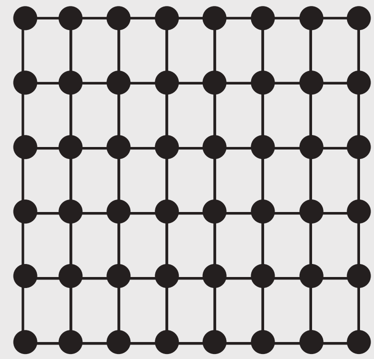
Here is an activity for better understanding of the inverse proportion.

DO THIS

Take a squared paper and arrange 48 counters on it in different number of rows as shown below.



4 Rows, 12 columns



6 Rows, 8 columns

Number of Rows (R)	(R_1) 2	(R_2) 3	(R_3) 4	(R_4) 6	(R_5) 8
Number of Columns (C)	(C_1) ...	(C_2) ...	(C_3) 12	(C_4) 8	(C_5) ...

What do you observe? As R increases, C decreases.

(i) Is $R_1 : R_2 = C_2 : C_1$? (ii) Is $R_3 : R_4 = C_4 : C_3$?

(iii) Are R and C inversely proportional to each other?

Try this activity with 36 counters.

TRY THESE

Observe the following tables and find which pair of variables (here x and y) are in inverse proportion.

(i)

x	50	40	30	20
y	5	6	7	8

(ii)

x	100	200	300	400
y	60	30	20	15

(iii)

x	90	60	45	30	20	5
y	10	15	20	25	30	35

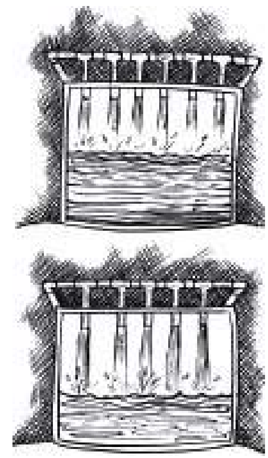
Let us consider some examples where we use the concept of inverse proportion.

When two quantities x and y are in direct proportion (or vary directly) they are also written as $x \propto y$.

When two quantities x and y are in inverse proportion (or vary inversely) they are also written as $x \propto \frac{1}{y}$.

Example 7: 6 pipes are required to fill a tank in 1 hour 20 minutes. How long will it take if only 5 pipes of the same type are used?

Solution:



Let the desired time to fill the tank be x minutes. Thus, we have the following table.

Number of pipes	6	5
Time (in minutes)	80	x

Lesser the number of pipes, more will be the time required by it to fill the tank. So, this is a case of inverse proportion.

Hence, $80 \times 6 = x \times 5$ $[x_1 y_1 = x_2 y_2]$

or $\frac{80 \times 6}{5} = x$

or $x = 96$

Thus, time taken to fill the tank by 5 pipes is 96 minutes or 1 hour 36 minutes.

Example 8: There are 100 students in a hostel. Food provision for them is for 20 days. How long will these provisions last, if 25 more students join the group?

Solution: Suppose the provisions last for y days when the number of students is 125. We have the following table.

Number of students	100	125
Number of days	20	y

Note that more the number of students, the sooner would the provisions exhaust. Therefore, this is a case of inverse proportion.

So, $100 \times 20 = 125 \times y$

or $\frac{100 \times 20}{125} = y$ or $16 = y$

Thus, the provisions will last for 16 days, if 25 more students join the hostel.

Alternately, we can write $x_1 y_1 = x_2 y_2$ as $\frac{x_1}{x_2} = \frac{y_2}{y_1}$.

That is, $x_1 : x_2 = y_2 : y_1$

or $100 : 125 = y : 20$

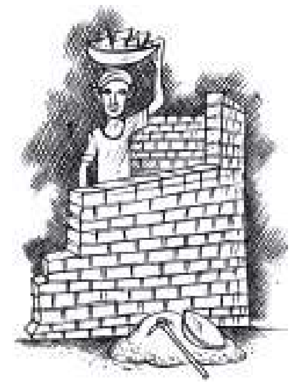
or $y = \frac{100 \times 20}{125} = 16$

Example 9: If 15 workers can build a wall in 48 hours, how many workers will be required to do the same work in 30 hours?

Solution:

Let the number of workers employed to build the wall in 30 hours be y.

We have the following table.



Number of hours	48	30
Number of workers	15	y

Obviously more the number of workers, faster will they build the wall.
So, the number of hours and number of workers vary in inverse proportion.

So $48 \times 15 = 30 \times y$

Therefore, $\frac{48 \times 15}{30} = y$ or $y = 24$

i.e., to finish the work in 30 hours, 24 workers are required.

EXERCISE 13.2

1. Which of the following are in inverse proportion?

(i) The number of workers on a job and the time to complete the job.

(ii) The time taken for a journey and the distance travelled in a uniform speed.

(iii) Area of cultivated land and the crop harvested.

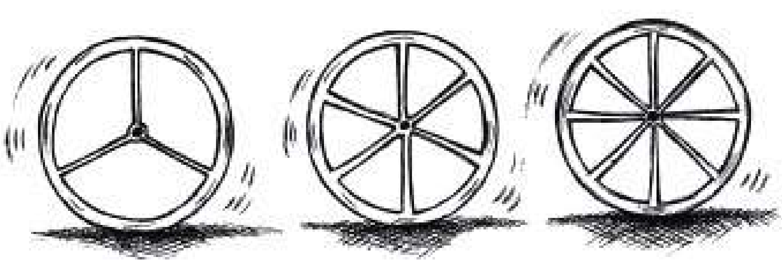
(iv) The time taken for a fixed journey and the speed of the vehicle.

(v) The population of a country and the area of land per person.

2. In a Television game show, the prize money of Rs 1,00,000 is to be divided equally amongst the winners. Complete the following table and find whether the prize money given to an individual winner is directly or inversely proportional to the number of winners?

Number of winners	1	2	4	5	8	10	20
Prize for each winner (in Rs)	1,00,000	50,000	...	...	...	...	...

3. Rehman is making a wheel using spokes. He wants to fix equal spokes in such a way that the angles between any pair of consecutive spokes are equal. Help him by completing the following table.



Number of spokes	4	6	8	10	12
Angle between a pair of consecutive spokes	90°	60°	...	...	...

(i) Are the number of spokes and the angles formed between the pairs of consecutive spokes in inverse proportion?

(ii) Calculate the angle between a pair of consecutive spokes on a wheel with 15 spokes.

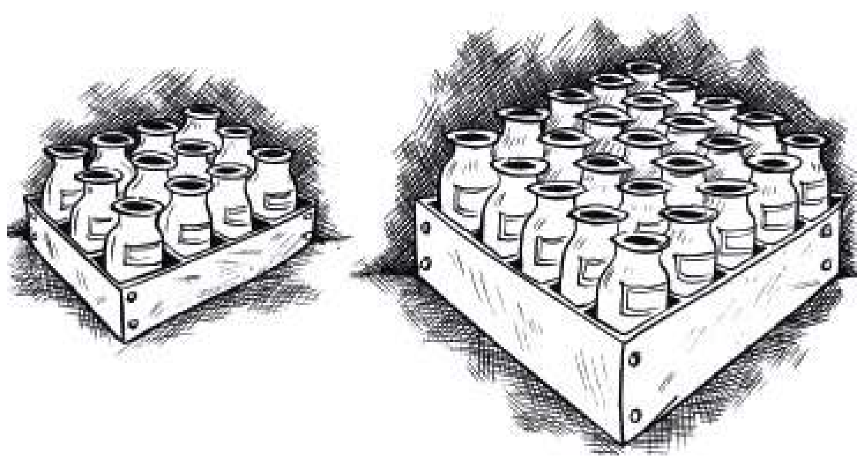
(iii) How many spokes would be needed, if the angle between a pair of consecutive spokes is 40°?

4. If a box of sweets is divided among 24 children, they will get 5 sweets each. How many would each get, if the number of the children is reduced by 4?

5. A farmer has enough food to feed 20 animals in his cattle for 6 days. How long would the food last if there were 10 more animals in his cattle?

6. A contractor estimates that 3 persons could rewire Jasminder's house in 4 days. If, he uses 4 persons instead of three, how long should they take to complete the job?

7. A batch of bottles were packed in 25 boxes with 12 bottles in each box. If the same batch is packed using 20 bottles in each box, how many boxes would be filled?



8. A factory requires 42 machines to produce a given number of articles in 63 days. How many machines would be required to produce the same number of articles in 54 days?

9. A car takes 2 hours to reach a destination by travelling at the speed of 60 km/h. How long will it take when the car travels at the speed of 80 km/h?

10. Two persons could fit new windows in a house in 3 days.

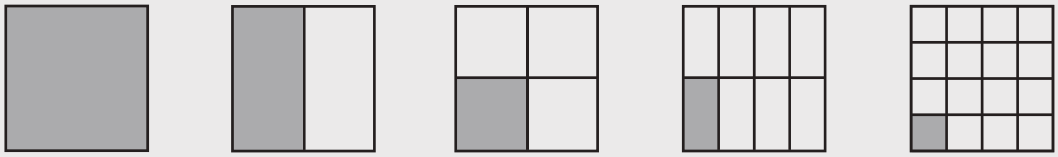
(i) One of the persons fell ill before the work started. How long would the job take now?

(ii) How many persons would be needed to fit the windows in one day?

11. A school has 8 periods a day each of 45 minutes duration. How long would each period be, if the school has 9 periods a day, assuming the number of school hours to be the same?

DO THIS

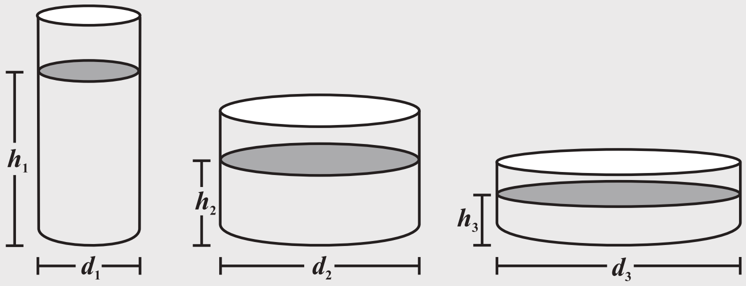
1. Take a sheet of paper. Fold it as shown in the figure. Count the number of parts and the area of a part in each case.



Tabulate your observations and discuss with your friends. Is it a case of inverse proportion? Why?

Number of parts	1	2	4	8	16
Area of each part	area of the paper	$\frac{1}{2}$ the area of the paper	...	...	...

2. Take a few containers of different sizes with circular bases. Fill the same amount of water in each container. Note the diameter of each container and the respective height at which the water level stands. Tabulate your observations. Is it a case of inverse proportion?



Diameter of container (in cm)			
Height of water level (in cm)			

WHAT HAVE WE DISCUSSED?

1. Two quantities x and y are said to be in **direct proportion** if they increase (decrease) together in such a manner that the ratio of their corresponding values remains constant. That is if $\frac{x}{y} = k$ [k is a positive number], then x and y are said to vary directly. In such a case if y_1, y_2 are the values of y corresponding to the values x_1, x_2 of x respectively then $\frac{x_1}{y_1} = \frac{x_2}{y_2}$.

2. Two quantities x and y are said to be in **inverse proportion** if an increase in x causes a proportional decrease in y (and vice-versa) in such a manner that the product of their corresponding values remains constant. That is, if $xy = k$, then x and y are said to vary inversely. In this case if y_1, y_2 are the values of y corresponding to the values x_1, x_2 of x respectively then $x_1 y_1 = x_2 y_2$ or $\frac{x_1}{x_2} = \frac{y_2}{y_1}$.