

## HERON'S FORMULA

### (A) Main Concepts and Results

- **Rectangle**

- (a) Area = length  $\times$  breadth
- (b) Perimeter = 2 (length + breadth)
- (c) Diagonal =  $\sqrt{(\text{length})^2 + (\text{breadth})^2}$

- **Square**

- (a) Area = (side)<sup>2</sup>
- (b) Perimeter = 4  $\times$  side
- (c) Diagonal =  $\sqrt{2}$   $\times$  side

- **Triangle with base ( $b$ ) and altitude ( $h$ )**

$$\text{Area} = \frac{1}{2} \times b \times h$$

- **Triangle with sides as  $a, b, c$**

- (i) Semi-perimeter =  $\frac{a + b + c}{2} = s$

- (ii) Area =  $\sqrt{s(s-a)(s-b)(s-c)}$  (Heron's Formula)

- **Isosceles triangle, with base  $a$  and equal sides  $b$**

$$\text{Area of isosceles triangle} = \frac{a}{4} \sqrt{4b^2 - a^2}$$

- **Equilateral triangle with side  $a$**

$$\text{Area} = \frac{\sqrt{3}}{4} a^2$$

- **Parallelogram with base  $b$  and altitude  $h$**

$$\text{Area} = bh$$

- **Rhombus with diagonals  $d_1$  and  $d_2$**

$$(a) \quad \text{Area} = \frac{1}{2} d_1 \times d_2$$

$$(b) \quad \text{Perimeter} = 2\sqrt{d_1^2 + d_2^2}$$

- **Trapezium with parallel sides  $a$  and  $b$ , and the distance between two parallel sides as  $h$ .**

$$\text{Area} = \frac{1}{2} (a + b) \times h$$

- **Regular hexagon with side  $a$**

$$\text{Area} = 6 \times \text{Area of an equilateral triangle with side } a$$

$$= 6 \times \frac{\sqrt{3}}{4} a^2 = \frac{3}{2} \sqrt{3} a^2$$

## (B) Multiple Choice Questions

**Write the correct answer:**

**Sample Question 1 :** The base of a right triangle is 8 cm and hypotenuse is 10 cm. Its area will be

- (A) 24 cm<sup>2</sup>      (B) 40 cm<sup>2</sup>      (C) 48 cm<sup>2</sup>      (D) 80 cm<sup>2</sup>

**Solution :** Answer (A)

## EXERCISE 12.1

1. An isosceles right triangle has area 8 cm<sup>2</sup>. The length of its hypotenuse is

- (A)  $\sqrt{32}$  cm      (B)  $\sqrt{16}$  cm      (C)  $\sqrt{48}$  cm      (D)  $\sqrt{24}$  cm

2. The perimeter of an equilateral triangle is 60 m. The area is  
(A)  $10\sqrt{3} \text{ m}^2$  (B)  $15\sqrt{3} \text{ m}^2$  (C)  $20\sqrt{3} \text{ m}^2$  (D)  $100\sqrt{3} \text{ m}^2$
3. The sides of a triangle are 56 cm, 60 cm and 52 cm long. Then the area of the triangle is  
(A)  $1322 \text{ cm}^2$  (B)  $1311 \text{ cm}^2$  (C)  $1344 \text{ cm}^2$  (D)  $1392 \text{ cm}^2$
4. The area of an equilateral triangle with side  $2\sqrt{3}$  cm is  
(A)  $5.196 \text{ cm}^2$  (B)  $0.866 \text{ cm}^2$  (C)  $3.496 \text{ cm}^2$  (D)  $1.732 \text{ cm}^2$
5. The length of each side of an equilateral triangle having an area of  $9\sqrt{3} \text{ cm}^2$  is  
(A) 8 cm (B) 36 cm (C) 4 cm (D) 6 cm
6. If the area of an equilateral triangle is  $16\sqrt{3} \text{ cm}^2$ , then the perimeter of the triangle is  
(A) 48 cm (B) 24 cm (C) 12 cm (D) 36 cm
7. The sides of a triangle are 35 cm, 54 cm and 61 cm, respectively. The length of its longest altitude  
(A)  $16\sqrt{5} \text{ cm}$  (B)  $10\sqrt{5} \text{ cm}$  (C)  $24\sqrt{5} \text{ cm}$  (D) 28 cm
8. The area of an isosceles triangle having base 2 cm and the length of one of the equal sides 4 cm, is  
(A)  $\sqrt{15} \text{ cm}^2$  (B)  $\sqrt{\frac{15}{2}} \text{ cm}^2$  (C)  $2\sqrt{15} \text{ cm}^2$  (D)  $4\sqrt{15} \text{ cm}^2$
9. The edges of a triangular board are 6 cm, 8 cm and 10 cm. The cost of painting it at the rate of 9 paise per  $\text{cm}^2$  is  
(A) Rs 2.00 (B) Rs 2.16 (C) Rs 2.48 (D) Rs 3.00

### (C) Short Answer Questions with Reasoning

Write **True** or **False** and justify your answer:

**Sample Question 1 :** If  $a, b, c$  are the lengths of three sides of a triangle, then area of a triangle =  $\sqrt{s(s-a)(s-b)(s-c)}$ , where  $s$  = perimeter of triangle.

**Solution :** False. Since in Heron's formula,

$$s = \frac{1}{2}(a+b+c)$$
$$= \frac{1}{2} (\text{perimeter of triangle})$$

### EXERCISE 12.2

Write **True** or **False** and justify your answer:

1. The area of a triangle with base 4 cm and height 6 cm is 24 cm<sup>2</sup>.
2. The area of  $\Delta ABC$  is 8 cm<sup>2</sup> in which  $AB = AC = 4$  cm and  $\angle A = 90^\circ$ .
3. The area of the isosceles triangle is  $\frac{5}{4}\sqrt{11}$  cm<sup>2</sup>, if the perimeter is 11 cm and the base is 5 cm.
4. The area of the equilateral triangle is  $20\sqrt{3}$  cm<sup>2</sup> whose each side is 8 cm.
5. If the side of a rhombus is 10 cm and one diagonal is 16 cm, the area of the rhombus is 96 cm<sup>2</sup>.
6. The base and the corresponding altitude of a parallelogram are 10 cm and 3.5 cm, respectively. The area of the parallelogram is 30 cm<sup>2</sup>.
7. The area of a regular hexagon of side 'a' is the sum of the areas of the five equilateral triangles with side a.
8. The cost of levelling the ground in the form of a triangle having the sides 51 m, 37 m and 20 m at the rate of Rs 3 per m<sup>2</sup> is Rs 918.
9. In a triangle, the sides are given as 11 cm, 12 cm and 13 cm. The length of the altitude is 10.25 cm corresponding to the side having length 12 cm.

#### (D) Short Answer Questions

**Sample Question 1 :** The sides of a triangular field are 41 m, 40 m and 9 m. Find the number of rose beds that can be prepared in the field, if each rose bed, on an average needs 900 cm<sup>2</sup> space.

**Solution :** Let  $a = 41$  m,  $b = 40$  m,  $c = 9$  m.

$$s = \frac{a+b+c}{2} = \frac{41+40+9}{2} \text{ m} = 45 \text{ m}$$

Area of the triangular field

$$\begin{aligned}
 &= \sqrt{s(s-a)(s-b)(s-c)} \\
 &= \sqrt{45(45-41)(45-40)(45-9)} \\
 &= \sqrt{45 \times 4 \times 5 \times 36} = 180 \text{ m}^2
 \end{aligned}$$

So, the number of rose beds =  $\frac{180}{0.09} = 2000$

**Sample Question 2 :** Calculate the area of the shaded region in Fig. 12.1.

**Solution :** For the triangle having the sides 122 m, 120 m and 22 m :

$$s = \frac{122 + 120 + 22}{2} = 132$$

$$\begin{aligned}
 \text{Area of the triangle} &= \sqrt{s(s-a)(s-b)(s-c)} \\
 &= \sqrt{132(132-122)(132-120)(132-22)} \\
 &= \sqrt{132 \times 10 \times 12 \times 110} \\
 &= 1320 \text{ m}^2
 \end{aligned}$$

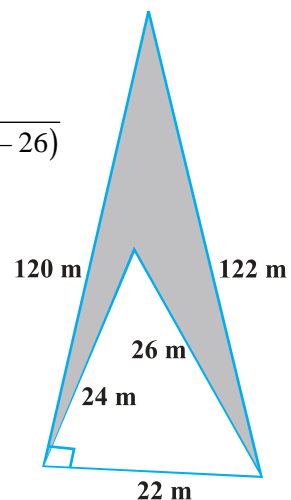
For the triangle having the sides 22 m, 24 m and 26 m:

$$s = \frac{22 + 24 + 26}{2} = 36$$

$$\begin{aligned}
 \text{Area of the triangle} &= \sqrt{36(36-22)(36-24)(36-26)} \\
 &= \sqrt{36 \times 14 \times 12 \times 10} \\
 &= 24\sqrt{105} \\
 &= 24 \times 10.25 \text{ m}^2 \text{ (approx.)} \\
 &= 246 \text{ m}^2
 \end{aligned}$$

Therefore, the area of the shaded portion

$$\begin{aligned}
 &= (1320 - 246) \text{ m}^2 \\
 &= 1074 \text{ m}^2
 \end{aligned}$$



**Fig. 12.1**

### EXERCISE 12.3

- Find the cost of laying grass in a triangular field of sides 50 m, 65 m and 65 m at the rate of Rs 7 per  $\text{m}^2$ .
- The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 13 m, 14 m and 15 m. The advertisements yield an earning of Rs 2000 per  $\text{m}^2$  a year. A company hired one of its walls for 6 months. How much rent did it pay?
- From a point in the interior of an equilateral triangle, perpendiculars are drawn on the three sides. The lengths of the perpendiculars are 14 cm, 10 cm and 6 cm. Find the area of the triangle.
- The perimeter of an isosceles triangle is 32 cm. The ratio of the equal side to its base is 3 : 2. Find the area of the triangle.
- Find the area of a parallelogram given in Fig. 12.2. Also find the length of the altitude from vertex A on the side DC.
- A field in the form of a parallelogram has sides 60 m and 40 m and one of its diagonals is 80 m long. Find the area of the parallelogram.
- The perimeter of a triangular field is 420 m and its sides are in the ratio 6 : 7 : 8. Find the area of the triangular field.
- The sides of a quadrilateral ABCD are 6 cm, 8 cm, 12 cm and 14 cm (taken in order) respectively, and the angle between the first two sides is a right angle. Find its area.
- A rhombus shaped sheet with perimeter 40 cm and one diagonal 12 cm, is painted on both sides at the rate of Rs 5 per  $\text{m}^2$ . Find the cost of painting.
- Find the area of the trapezium PQRS with height PQ given in Fig. 12.3

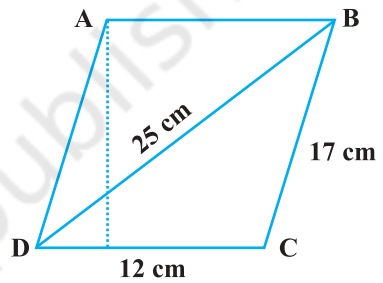


Fig. 12.2

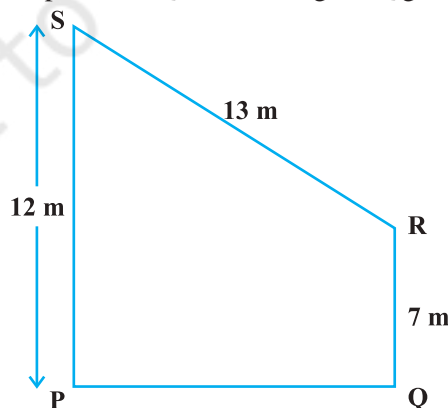


Fig. 12.3

### (E) Long Answer Questions

**Sample Question 1 :** If each side of a triangle is doubled, then find the ratio of area of the new triangle thus formed and the given triangle.

**Solution :** Let  $a, b, c$  be the sides of the triangle (existing) and  $s$  be its semi-perimeter.

$$\text{Then, } s = \frac{a+b+c}{2}$$

$$\text{or, } 2s = a + b + c \tag{1}$$

$$\text{Area of the existing triangle} = \sqrt{s(s-a)(s-b)(s-c)} = \Delta, \text{ say}$$

According to the statement, the sides of the new triangle will be  $2a, 2b$  and  $2c$ . Let  $S$  be the semi-perimeter of the new triangle.

$$S = \frac{2a+2b+2c}{2} = a+b+c \tag{2}$$

From (1) and (2), we get

$$S = 2s \tag{3}$$

Area of the new triangle

$$= \sqrt{S(S-2a)(S-2b)(S-2c)}$$

Putting the values, we get

$$= \sqrt{2s(2s-2a)(2s-2b)(2s-2c)}$$

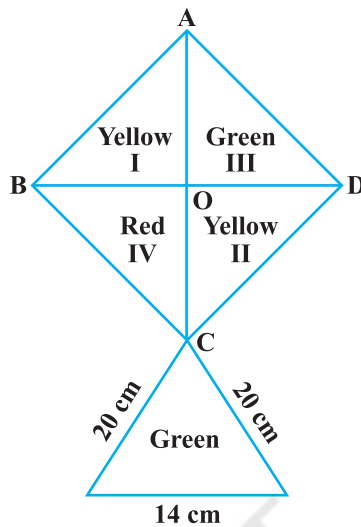
$$= \sqrt{16s(s-a)(s-b)(s-c)}$$

$$= 4\sqrt{s(s-a)(s-b)(s-c)} = 4\Delta$$

Therefore, the required ratio is 4:1.

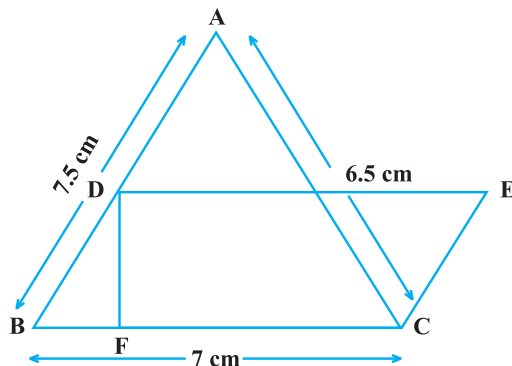
### EXERCISE 12.4

1. How much paper of each shade is needed to make a kite given in Fig. 12.4, in which ABCD is a square with diagonal 44 cm.



**Fig. 12.4**

2. The perimeter of a triangle is 50 cm. One side of a triangle is 4 cm longer than the smaller side and the third side is 6 cm less than twice the smaller side. Find the area of the triangle.
3. The area of a trapezium is  $475 \text{ cm}^2$  and the height is 19 cm. Find the lengths of its two parallel sides if one side is 4 cm greater than the other.
4. A rectangular plot is given for constructing a house, having a measurement of 40 m long and 15 m in the front. According to the laws, a minimum of 3 m, wide space should be left in the front and back each and 2 m wide space on each of other sides. Find the largest area where house can be constructed.
5. A field is in the shape of a trapezium having parallel sides 90 m and 30 m. These sides meet the third side at right angles. The length of the fourth side is 100 m. If it costs Rs 4 to plough  $1 \text{ m}^2$  of the field, find the total cost of ploughing the field.
6. In Fig. 12.5,  $\triangle ABC$  has sides  $AB = 7.5 \text{ cm}$ ,  $AC = 6.5 \text{ cm}$  and  $BC = 7 \text{ cm}$ . On base BC a parallelogram DBCE of same area as that of  $\triangle ABC$  is constructed. Find the height DF of the parallelogram.
7. The dimensions of a rectangle ABCD are  $51 \text{ cm} \times 25 \text{ cm}$ . A trapezium PQCD with its parallel



**Fig. 12.5**



sides QC and PD in the ratio 9 : 8, is cut off from the rectangle as shown in the Fig. 12.6. If the area of the trapezium PQCD is  $\frac{5}{6}$ th part of the area of the rectangle, find the lengths QC and PD.

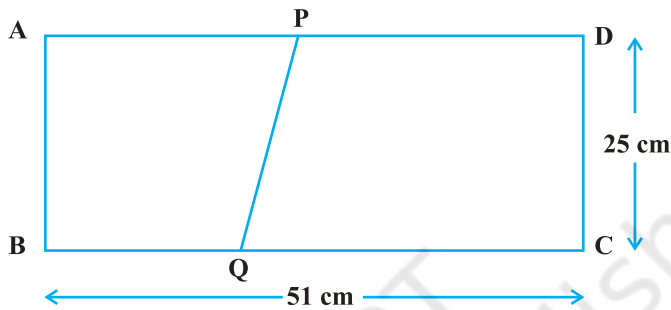


Fig. 12.6

8. A design is made on a rectangular tile of dimensions 50 cm × 70 cm as shown in Fig. 12.7. The design shows 8 triangles, each of sides 26 cm, 17 cm and 25 cm. Find the total area of the design and the remaining area of the tile.

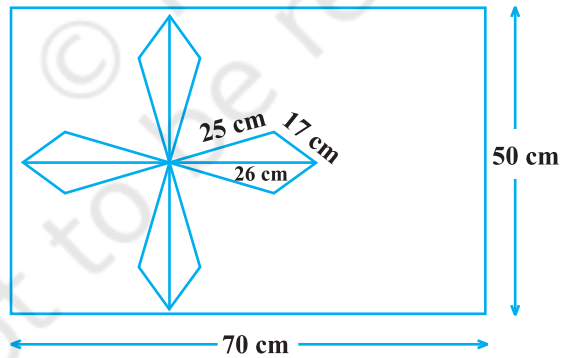


Fig. 12.7