

TRIANGLES

(A) Main Concepts and Results

Triangles and their parts, Congruence of triangles, Congruence and correspondence of vertices, Criteria for Congruence of triangles: (i) SAS (ii) ASA (iii) SSS (iv) RHS

AAS criterion for congruence of triangles as a particular case of ASA criterion.

- Angles opposite to equal sides of a triangle are equal,
- Sides opposite to equal angles of a triangle are equal,
- A point equidistant from two given points lies on the perpendicular bisector of the line-segment joining the two points and its converse,
- A point equidistant from two intersecting lines lies on the bisectors of the angles formed by the two lines,
- In a triangle
 - (i) side opposite to the greater angle is longer
 - (ii) angle opposite the longer side is greater
 - (iii) the sum of any two sides is greater than the third side.

(B) Multiple Choice Questions

Write the correct answer :

Sample Question 1 : If $\triangle ABC \cong \triangle PQR$ and $\triangle ABC$ is not congruent to $\triangle RPQ$, then which of the following is not true:

- (A) $BC = PQ$ (B) $AC = PR$ (C) $QR = BC$ (D) $AB = PQ$

Solution : Answer (A)

EXERCISE 7.1

In each of the following, write the correct answer:

- Which of the following is not a criterion for congruence of triangles?
(A) SAS (B) ASA (C) SSA (D) SSS
- If $AB = QR$, $BC = PR$ and $CA = PQ$, then
(A) $\triangle ABC \cong \triangle PQR$ (B) $\triangle CBA \cong \triangle PRQ$
(C) $\triangle BAC \cong \triangle RPQ$ (D) $\triangle PQR \cong \triangle BCA$
- In $\triangle ABC$, $AB = AC$ and $\angle B = 50^\circ$. Then $\angle C$ is equal to
(A) 40° (B) 50° (C) 80° (D) 130°
- In $\triangle ABC$, $BC = AB$ and $\angle B = 80^\circ$. Then $\angle A$ is equal to
(A) 80° (B) 40° (C) 50° (D) 100°
- In $\triangle PQR$, $\angle R = \angle P$ and $QR = 4$ cm and $PR = 5$ cm. Then the length of PQ is
(A) 4 cm (B) 5 cm (C) 2 cm (D) 2.5 cm
- D is a point on the side BC of a $\triangle ABC$ such that AD bisects $\angle BAC$. Then
(A) $BD = CD$ (B) $BA > BD$ (C) $BD > BA$ (D) $CD > CA$
- It is given that $\triangle ABC \cong \triangle FDE$ and $AB = 5$ cm, $\angle B = 40^\circ$ and $\angle A = 80^\circ$. Then which of the following is true?
(A) $DF = 5$ cm, $\angle F = 60^\circ$ (B) $DF = 5$ cm, $\angle E = 60^\circ$
(C) $DE = 5$ cm, $\angle E = 60^\circ$ (D) $DE = 5$ cm, $\angle D = 40^\circ$
- Two sides of a triangle are of lengths 5 cm and 1.5 cm. The length of the third side of the triangle cannot be
(A) 3.6 cm (B) 4.1 cm (C) 3.8 cm (D) 3.4 cm
- In $\triangle PQR$, if $\angle R > \angle Q$, then
(A) $QR > PR$ (B) $PQ > PR$ (C) $PQ < PR$ (D) $QR < PR$
- In triangles ABC and PQR, $AB = AC$, $\angle C = \angle P$ and $\angle B = \angle Q$. The two triangles are
(A) isosceles but not congruent (B) isosceles and congruent
(C) congruent but not isosceles (D) neither congruent nor isosceles
- In triangles ABC and DEF, $AB = FD$ and $\angle A = \angle D$. The two triangles will be congruent by SAS axiom if
(A) $BC = EF$ (B) $AC = DE$ (C) $AC = EF$ (D) $BC = DE$

(C) Short Answer Questions with Reasoning

Sample Question 1: In the two triangles ABC and DEF, $AB = DE$ and $AC = EF$. Name two angles from the two triangles that must be equal so that the two triangles are congruent. Give reason for your answer.

Solution: The required two angles are $\angle A$ and $\angle E$. When $\angle A = \angle E$, $\triangle ABC \cong \triangle EDF$ by SAS criterion.

Sample Question 2: In triangles ABC and DEF, $\angle A = \angle D$, $\angle B = \angle E$ and $AB = EF$. Will the two triangles be congruent? Give reasons for your answer.

Solution: Two triangles need not be congruent, because AB and EF are not corresponding sides in the two triangles.

EXERCISE 7.2

1. In triangles ABC and PQR, $\angle A = \angle Q$ and $\angle B = \angle R$. Which side of $\triangle PQR$ should be equal to side AB of $\triangle ABC$ so that the two triangles are congruent? Give reason for your answer.
2. In triangles ABC and PQR, $\angle A = \angle Q$ and $\angle B = \angle R$. Which side of $\triangle PQR$ should be equal to side BC of $\triangle ABC$ so that the two triangles are congruent? Give reason for your answer.
3. "If two sides and an angle of one triangle are equal to two sides and an angle of another triangle, then the two triangles must be congruent." Is the statement true? Why?
4. "If two angles and a side of one triangle are equal to two angles and a side of another triangle, then the two triangles must be congruent." Is the statement true? Why?
5. Is it possible to construct a triangle with lengths of its sides as 4 cm, 3 cm and 7 cm? Give reason for your answer.
6. It is given that $\triangle ABC \cong \triangle RPQ$. Is it true to say that $BC = QR$? Why?
7. If $\triangle PQR \cong \triangle EDF$, then is it true to say that $PR = EF$? Give reason for your answer.
8. In $\triangle PQR$, $\angle P = 70^\circ$ and $\angle R = 30^\circ$. Which side of this triangle is the longest? Give reason for your answer.
9. AD is a median of the triangle ABC. Is it true that $AB + BC + CA > 2 AD$? Give reason for your answer.
10. M is a point on side BC of a triangle ABC such that AM is the bisector of $\angle BAC$. Is it true to say that perimeter of the triangle is greater than $2 AM$? Give reason for your answer.

11. Is it possible to construct a triangle with lengths of its sides as 9 cm, 7 cm and 17 cm? Give reason for your answer.
12. Is it possible to construct a triangle with lengths of its sides as 8 cm, 7 cm and 4 cm? Give reason for your answer.

(D) Short Answer Questions

Sample Question 1 : In Fig 7.1, $PQ = PR$ and $\angle Q = \angle R$. Prove that $\Delta PQS \cong \Delta PRT$.

Solution : In ΔPQS and ΔPRT ,

$$PQ = PR \quad (\text{Given})$$

$$\angle Q = \angle R \quad (\text{Given})$$

and $\angle QPS = \angle RPT$ (Same angle)

Therefore, $\Delta PQS \cong \Delta PRT$ (ASA)

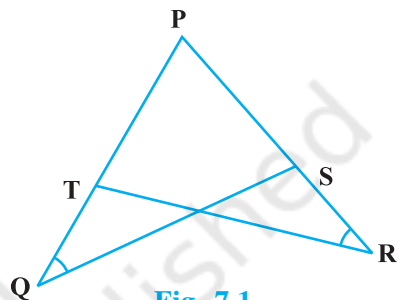


Fig. 7.1

Sample Question 2 : In Fig.7.2, two lines AB and CD intersect each other at the point O such that $BC \parallel DA$ and $BC = DA$. Show that O is the mid-point of both the line-segments AB and CD.

Solution : $BC \parallel AD$ (Given)

Therefore, $\angle CBO = \angle DAO$ (Alternate interior angles)

and $\angle BCO = \angle ADO$ (Alternate interior angles)

Also, $BC = DA$ (Given)

So, $\Delta BOC \cong \Delta AOD$ (ASA)

Therefore, $OB = OA$ and $OC = OD$, i.e., O is the mid-point of both AB and CD.

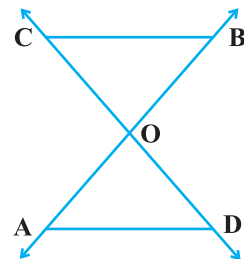


Fig. 7.2

Sample Question 3 : In Fig.7.3, $PQ > PR$ and QS and RS are the bisectors of $\angle Q$ and $\angle R$, respectively. Show that $SQ > SR$.

Solution : $PQ > PR$ (Given)

Therefore, $\angle R > \angle Q$ (Angles opposite the longer side is greater)

So, $\angle SRQ > \angle SQR$ (Half of each angle)

Therefore, $SQ > SR$ (Side opposite the greater angle will be longer)

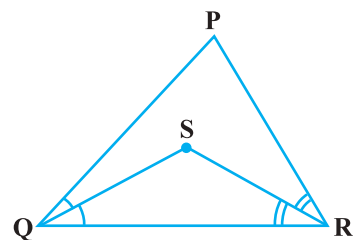


Fig. 7.3

EXERCISE 7.3

1. ABC is an isosceles triangle with $AB = AC$ and BD and CE are its two medians. Show that $BD = CE$.
2. In Fig.7.4, D and E are points on side BC of a $\triangle ABC$ such that $BD = CE$ and $AD = AE$. Show that $\triangle ABD \cong \triangle ACE$.
3. CDE is an equilateral triangle formed on a side CD of a square $ABCD$ (Fig.7.5). Show that $\triangle ADE \cong \triangle BCE$.

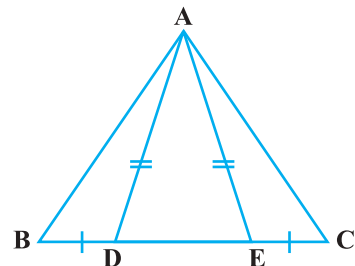


Fig. 7.4

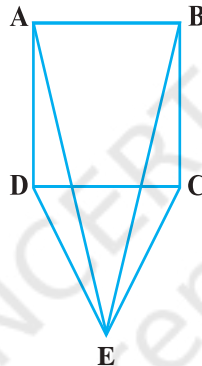


Fig. 7.5

4. In Fig.7.6, $BA \perp AC$, $DE \perp DF$ such that $BA = DE$ and $BF = EC$. Show that $\triangle ABC \cong \triangle DEF$.
5. Q is a point on the side SR of a $\triangle PSR$ such that $PQ = PR$. Prove that $PS > PQ$.
6. S is any point on side QR of a $\triangle PQR$. Show that: $PQ + QR + RP > 2 PS$.
7. D is any point on side AC of a $\triangle ABC$ with $AB = AC$. Show that $CD < BD$.
8. In Fig. 7.7, $l \parallel m$ and M is the mid-point of a line segment AB . Show that M is also the mid-point of any line segment CD , having its end points on l and m , respectively.
9. Bisectors of the angles B and C of an isosceles triangle with $AB = AC$ intersect each other at O . BO is produced to a point M . Prove that $\angle MOC = \angle ABC$.

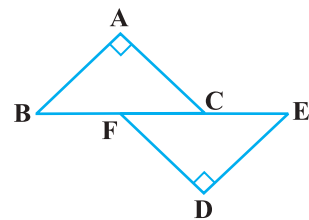


Fig. 7.6

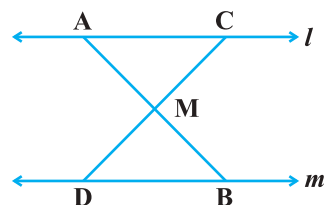


Fig. 7.7

10. Bisectors of the angles B and C of an isosceles triangle ABC with $AB = AC$ intersect each other at O. Show that external angle adjacent to $\angle ABC$ is equal to $\angle BOC$.
11. In Fig. 7.8, AD is the bisector of $\angle BAC$. Prove that $AB > BD$.

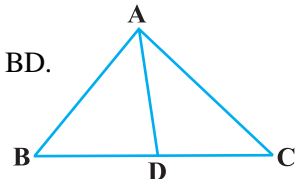


Fig. 7.8

(E) Long Answer Questions

Sample Question 1: In Fig. 7.9, ABC is a right triangle and right angled at B such that $\angle BCA = 2 \angle BAC$. Show that hypotenuse $AC = 2 BC$.

Solution: Produce CB to a point D such that $BC = BD$ and join AD.

In $\triangle ABC$ and $\triangle ABD$, we have

$$\begin{aligned} BC &= BD && \text{(By construction)} \\ AB &= AB && \text{(Same side)} \\ \angle ABC &= \angle ABD && \text{(Each of } 90^\circ) \end{aligned}$$

Therefore, $\triangle ABC \cong \triangle ABD$ (SAS)

So, $\angle CAB = \angle DAB$ } (CPCT) (1)

and $AC = AD$ } (2)

Thus, $\angle CAD = \angle CAB + \angle BAD = x + x = 2x$ [From (1)] (3)

and $\angle ACD = \angle ADB = 2x$ [From (2), $AC = AD$] (4)

That is, $\triangle ACD$ is an equilateral triangle. [From (3) and (4)]

or $AC = CD$, i.e., $AC = 2 BC$ (Since $BC = BD$)

Sample Question 2 : Prove that if in two triangles two angles and the included side of one triangle are equal to two angles and the included side of the other triangle, then the two triangles are congruent.

Solution: See proof of Theorem 7.1 of Class IX Mathematics Textbook.

Sample Question 3 : If the bisector of an angle of a triangle also bisects the opposite side, prove that the triangle is isosceles.

Solution : We are given a point D on side BC of a $\triangle ABC$ such that $\angle BAD = \angle CAD$ and $BD = CD$ (see Fig. 7.10). We are to prove that $AB = AC$.

Produce AD to a point E such that $AD = DE$ and then join CE.

Now, in $\triangle ABD$ and $\triangle ECD$, we have

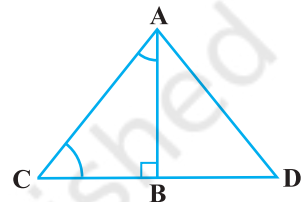


Fig. 7.9

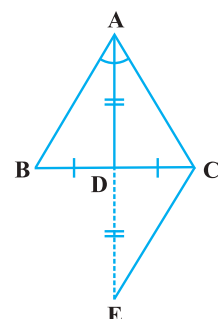


Fig. 7.10

$BD = CD$ (Given)

$AD = ED$ (By construction)

and $\angle ADB = \angle EDC$ (Vertically opposite angles)

Therefore, $\triangle ABD \cong \triangle ECD$ (SAS)

So, $AB = EC$ (1)

and $\angle BAD = \angle CED$ (2) (CPCT)

Also, $\angle BAD = \angle CAD$ (Given)

Therefore, $\angle CAD = \angle CED$ [From (2)]

So, $AC = EC$ [Sides opposite the equal angles] (3)

Therefore, $AB = AC$ [From (1) and (3)]

Sample Question 4 : S is any point in the interior of $\triangle PQR$. Show that $SQ + SR < PQ + PR$.

Solution : Produce QS to intersect PR at T (See Fig. 7.11).

From $\triangle PQT$, we have

$PQ + PT > QT$ (Sum of any two sides is greater than the third side)

i.e., $PQ + PT > SQ + ST$ (1)

From $\triangle TSR$, we have

$ST + TR > SR$ (2)

Adding (1) and (2), we get

$PQ + PT + ST + TR > SQ + ST + SR$

i.e., $PQ + PT + TR > SQ + SR$

i.e., $PQ + PR > SQ + SR$

or $SQ + SR < PQ + PR$

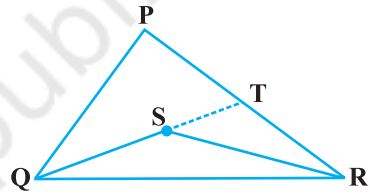


Fig. 7.11

EXERCISE 7.4

1. Find all the angles of an equilateral triangle.
2. The image of an object placed at a point A before a plane mirror LM is seen at the point B by an observer at D as shown in Fig. 7.12. Prove that the image is as far behind the mirror as the object is in front of the mirror.

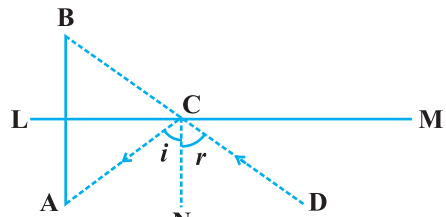


Fig. 7.12

[Hint: CN is normal to the mirror. Also, angle of incidence = angle of reflection].

3. ABC is an isosceles triangle with $AB = AC$ and D is a point on BC such that $AD \perp BC$ (Fig. 7.13). To prove that $\angle BAD = \angle CAD$, a student proceeded as follows:

In $\triangle ABD$ and $\triangle ACD$,

$$AB = AC \quad (\text{Given})$$

$$\angle B = \angle C \quad (\text{because } AB = AC)$$

and $\angle ADB = \angle ADC$

Therefore, $\triangle ABD \cong \triangle ACD$ (AAS)

So, $\angle BAD = \angle CAD$ (CPCT)

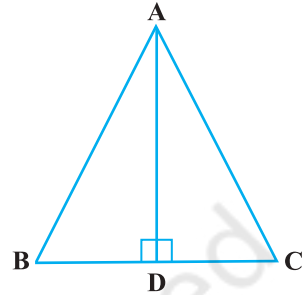


Fig. 7.13

What is the defect in the above arguments?

[Hint: Recall how $\angle B = \angle C$ is proved when $AB = AC$].

4. P is a point on the bisector of $\angle ABC$. If the line through P, parallel to BA meet BC at Q, prove that BPQ is an isosceles triangle.
5. ABCD is a quadrilateral in which $AB = BC$ and $AD = CD$. Show that BD bisects both the angles ABC and ADC.
6. ABC is a right triangle with $AB = AC$. Bisector of $\angle A$ meets BC at D. Prove that $BC = 2 AD$.
7. O is a point in the interior of a square ABCD such that OAB is an equilateral triangle. Show that $\triangle OCD$ is an isosceles triangle.
8. ABC and DBC are two triangles on the same base BC such that A and D lie on the opposite sides of BC, $AB = AC$ and $DB = DC$. Show that AD is the perpendicular bisector of BC.
9. ABC is an isosceles triangle in which $AC = BC$. AD and BE are respectively two altitudes to sides BC and AC. Prove that $AE = BD$.
10. Prove that sum of any two sides of a triangle is greater than twice the median with respect to the third side.
11. Show that in a quadrilateral ABCD, $AB + BC + CD + DA < 2 (BD + AC)$
12. Show that in a quadrilateral ABCD,
 $AB + BC + CD + DA > AC + BD$
13. In a triangle ABC, D is the mid-point of side AC such that $BD = \frac{1}{2} AC$. Show that $\angle ABC$ is a right angle.
14. In a right triangle, prove that the line-segment joining the mid-point of the hypotenuse to the opposite vertex is half the hypotenuse.

15. Two lines l and m intersect at the point O and P is a point on a line n passing through the point O such that P is equidistant from l and m . Prove that n is the bisector of the angle formed by l and m .
16. Line segment joining the mid-points M and N of parallel sides AB and DC , respectively of a trapezium $ABCD$ is perpendicular to both the sides AB and DC . Prove that $AD = BC$.
17. $ABCD$ is a quadrilateral such that diagonal AC bisects the angles A and C . Prove that $AB = AD$ and $CB = CD$.
18. ABC is a right triangle such that $AB = AC$ and bisector of angle C intersects the side AB at D . Prove that $AC + AD = BC$.
19. AB and CD are the smallest and largest sides of a quadrilateral $ABCD$. Out of $\angle B$ and $\angle D$ decide which is greater.
20. Prove that in a triangle, other than an equilateral triangle, angle opposite the longest side is greater than $\frac{2}{3}$ of a right angle.
21. $ABCD$ is quadrilateral such that $AB = AD$ and $CB = CD$. Prove that AC is the perpendicular bisector of BD .