ITL PUBLIC SCHOOL
ANNUAL EXAMINATION(2022-23)(Answer key)

Date:10.02 .23 **Class: XI**

MATHEMATICS(041) – SET A

Time: 3 hrs **M. M: 80**

	SECTION A Each question carries 1 mark	
1	Let A and B be two sets having 4 and 7 elements respectively. Then write the maximum number of elements that $A \cup B$ can have. **11**	1
2	If p, q be two A.M.'s and G be one G.M. between two numbers, then write G^2 in terms of p and q only. **(2p-q)(2q-p)**	1
3	Let f(x) be a function defined by $f(x) = \begin{cases} 4x-5, & \text{if } x \le 2 \\ x-\lambda, & \text{if } x > 2 \end{cases}$. Find λ, if $\lim_{x \to 2} f(x)$ exists. **-1**	1
4	If f(1) = 1, $f'(1) = 2$, then write the value of $\lim_{x \to 1} \frac{\sqrt{f(x)}-1}{\sqrt{x}-1}$. **=2**	1
5	Write the least positive integral value of n for which $\left(\frac{1+i}{1-i}\right)^n$ is equal to 1. **4**	1
6	What is the probability that a randomly chosen two digit positive integer is a multiple of 3? **30/90**	1
7	Find the value of $sin^2 75° + sin^2 15°$ **1**	1
8	If n is any positive integer, write the value of $\frac{i^{4n+1} - i^{4n-1}}{2}$ **i**	1
9	Expand using binomial theorem : $\left(x + \frac{2}{x}\right)^4$ $x^4 + \frac{16}{x^4} + 8x^2 + 24 + \frac{32}{x^2}$	1
10	Write the set X = {1,1/4,1/9,1/16,1/25,…..} in set builder form. $1/n^2$, n$\in N$	1
11	Solve the following in equations: $\frac{5x-2}{3} - \frac{7x-3}{5} > \frac{x}{4}$ **(4,∞)**	1
12	If ${}^{43}C_{r-6} = {}^{43}C_{3r+1}$, then find the value of r. **r = 12**	1
13	Find the range of the function $f(x) = \frac{x^2-9}{x-3}$ **R-{6}**	1
14	Find the eccentricity of the hyperbola satisfying the given conditions vertices (0,±3), Length of conjugate axis is 6. $\sqrt{2}$	1
15	Find the value of λ, if the lines 3x – 4y – 13 = 0, 8x – 11y – 33 = 0 and 2x – 3y + λ = 0 are concurrent. **-7**	1
16	Find the image of (- 2,3,4) in the yz - plane. **(2,3,4)**	1
17	Find the value of $\tan \frac{11\pi}{6}$ **-1/$\sqrt{3}$**	1
18	Find the distances of the point P (- 4,3,5) from y axis. $\sqrt{41}$	1
19	ASSERTION-REASON BASED QUESTIONS(19,20) In the following questions, a statement of assertion (A) is followed by a statement of	1

	Reason (R). Choose the correct answer out of the following choices. (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true. ASSERTION: The number of terms in the expansion of$\{(3x+y)^8-(3x-y)^8\}$ are 4. REASON: If n is even then the expansion of $\{(x+a)^n-(x-a)^n\}$ have $(n+2)/2$ terms. **(c)**	
20	Assertion (A) The fourth term of a GP is the square of its second term and the first term is −3, then its 7th term is equal to - 2187. Reason (R) : the nth term of G.P is $a\,r^{n-1}$ **(a)**	1
	SECTION B **This section comprises of very short answer type-questions (VSA) of 2 marks each**	
21	Find the equation of the line mid-way between the parallel lines 9x + 6y -7 = 0 and 3x + 2y + 6 = 0 The equations of the lines are $3x+2y-\frac{7}{3}=0$ ....(1) $3x+2y+6=0$ ....(2) Let the equation of the line mid-way between the parallel lines 1 and 2 be $3x+2y+\lambda=0$ ....(3) Then, Distance between the lines 1 and 3 = Distance between the lines 2 and 3 $\frac{\lvert\lambda+\frac{7}{3}\rvert}{\sqrt{9+4}}=\frac{\lvert\lambda-6\rvert}{\sqrt{9+4}}$ $\lvert\lambda+\frac{7}{3}\rvert=\lvert\lambda-6\rvert$ $\lambda+\frac{7}{3}=-\lambda+6$ $\lvert\lambda+\frac{7}{3}\rvert=\lvert\lambda-6\rvert$ $\lambda+\frac{7}{3}=-\lambda+6$ $2\lambda=\frac{11}{3}$ $\lambda=\frac{11}{6}$ Hence, the equation of the required line is $3x+2y+\frac{11}{6}=0$.	2
22	Using binomial theorem, prove that 6^n - 5n always leaves the remainder 1 when divided by 25. **OR** If a and b are distinct integers, prove that a^n -b^n is divisible by (a - b), whenever n ∈ N. Writing $6^n=(1+5)^n$ We know that $(a+b)^n={}^nC_0a^nb^0+{}^nC_1a^{n-1}b^1+.....+{}^nC_na^{n-n}b^n$ Putting a = 1, b = 5 $(6)^n={}^nC_01^n5^0+{}^nC_11^{n-1}5^1+{}^nC_21^{n-2}5^2+....+{}^nC_n1^{n-n}5^n$ $={}^nC_05^0+{}^nC_15^1+{}^nC_25^2+....+{}^nC_n5^n$ $=1\times1+\frac{n!}{1!(n-1)!}5^1+\frac{n!}{2!(n-2)!}5^2+....+1\times5^n$ $=1+\frac{n(n-1)!}{1!(n-1)!}5^1+\frac{n(n-1)(n-2)!}{2!(n-2)!}5^2+....+1\times5^n$ $=1+n(5)+\frac{n(n-1)}{2}5^2+....+5^n$ Thus, $(6)^n=1+5n+\frac{n(n-1)}{2}5^2+....+5^n$ $(6)^n-5n=1+\frac{n(n-1)}{2}5^2+....+5^n$ $(6)^n-5n=1+\frac{n(n-1)}{2}5^2+....+5^n$ $(6)^n-5n=1+5^2\left(\frac{n(n-1)}{2}+....+5^{n-2}\right)$ $(6)^n-5n=1+25\left(\frac{n(n-1)}{2}+....+5^{n-2}\right)$ $(6)^n-5n=1+25k$ where $k=\frac{n(n-1)}{2}+....+5^{n-2}$ The above equation is of the form Dividend = Divisor × Quotient + Remainder $6^n-5n=25k+1$ Hence 6^n-5n always leave remainder 1 when dividing by 25. **OR**	2

	It can be written that a = a - b + b $\therefore a^n$ = \left (a - b + b \right)^{n} = $[(a-b)+b]^n$ $= {}^nC_0(a-b)^n + {}^nC_2(a-b)^{n-1}b + \ldots\ldots + {}^nC_{n-1}(a-b)b^{n-1} + {}^nC_nb^n$ $= (a-b)^n + {}^nC_2(a-b)b^{n-1}b + \ldots. + {}^nC_{n-1}(a-b)b^{n-1} + b^n$ $\Rightarrow a^n - b^n = (a-b)\left[(a-b)^{n-1} + {}^nC_2(a-b)b^{n-2}b + \ldots. + {}^nC_{n-1}b^{n-1}\right]$ $a^n - b^n = k(a-b)$ $\Rightarrow a^n - b^n = k(a-b)$ where, $k = \left[(a-b)^{n-1} + {}^nC_2(a-b)b^{n-2}b + \ldots. + {}^nC_{n-1}b^{n-1}\right]$ is a natural number This shows that $(a-b)$ is a factor of $(a^n - b^n)$ where n is a positive integer.	
23	Evaluate: $\lim_{x\to 5}\frac{x-5}{\sqrt{6x-5}-\sqrt{4x+5}}$ **5** **OR** Find the value of k, if $\lim_{x\to 1}\frac{x^4-1}{x-1}=\lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}$. **k= 8/3**	2
24	If α and β are different complex numbers with $\|\beta\| = 1$, find $\left\|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right\|$. **OR** Find real value of x and y for which the following equalities hold: $(1+i)y^2+(6+i)=(2+i)x$ Ans: 1 **OR** x = 5, y = 2 or x =5, y = -2	2
25	Show that the points A (1, 3, 4), B (- 1, 6,10), C (- 7, 4, 7) and D (- 5, 1 ,1) are the vertices of a rhombus. Ans: All sides equal to 7 The distance between the points A (1, 3, 4) and C (-7, 4, 7) is AC, $= \sqrt{(1-(-7))^2+(3-4)^2+(4-7)^2}$ $= \sqrt{8^2+(-1)^2+(-3)^2}$ $= \sqrt{64+1+9}$ $= \sqrt{74}$ The distance between the points B (-1, 6, 10) and D (-5, 1, 1) is BD $= \sqrt{(-1-(-5))^2+(6-1)^2+(10-1)^2}$ $= \sqrt{4^2+5^2+9^2}$ $= \sqrt{16+25+81}$ $= \sqrt{112}$ $= 4\sqrt{7}$ It is clear that, AC ≠ BD The diagonals are not equal but all sides are equal.	2
	SECTION C **(This section comprises of short answer type questions (SA) of 3 marks each)**	
26	Evaluate: (a) $\lim_{x\to 0}\frac{\cot 2x - \mathrm{cosec}\, 2x}{x}$ (b) $\lim_{x\to 0}\frac{\sin 2x+\sin 3x}{2x+\sin 3x}$ Ans: **(a) -1 (b) 1**	1.5+1.5
27	a) Redefine the function: f(x) = |x – 1| + |x + 6|. b) Let A = {1,2,3,4,5,6}. Let R be a relation on A defined by {(a, b): a,b ∈ A, b is exactly divisible by a} (i) Write R in roster form (ii) Find the range of R. Ans: a) f(x) = $\begin{cases} -2x-5 & x<-6 \\ 7 & -6\le x<1 \\ 2x+5 & x>1 \end{cases}$	1.5+1.5

	c) Range of R={1,2,3,4,5,6} (i) R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)} - (5,5)	
28	How many litres of water will have to be added to 1125 litres of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content? **OR** a) A man wants to cut three lengths from a single piece of board of length 100 cm. The second length is to be 5 cm longer than the shortest and third length is to be twice as long as the shortest. What are the possible lengths for the shortest board if third piece is to be at least 5 cm longer than the second. b) Solve: $\lvert 3x-2 \rvert \le \frac{1}{2}$ Let the quantity of water to be added to solution =x liters $\therefore$ 25%(1125 + x) < 45% of 1125 $\Rightarrow \frac{25}{100}(1125+x) < \frac{45}{100} \times 1125$ $\Rightarrow 1125 + x < \frac{45}{100} \times 1125$ $\Rightarrow 1125 + x < 45 \times 45$ $\Rightarrow 1125 + x < 2025$ $\Rightarrow x < 2025 - 1125$ $\Rightarrow x < 900$ and 45% of 1125 < 30%(1125 + x) $\Rightarrow \frac{45}{100} \times 1125 < \frac{30}{100}(1125+x)$ $\Rightarrow \frac{45}{30} \times 1125 < 1125 + x$ $\Rightarrow \frac{3}{2} \times 1125 < 1125 + x$ $\Rightarrow 1.5 \times 1125 < 1125 + x$ $\Rightarrow 1687.5 < 1125 + x$ $\Rightarrow 168.75 - 1125 < x$ $\Rightarrow 562.5 < x$ ...(ii) Using (i) and (ii), we get 562.5<x<900 Hence,quantity of water lies between 562.5 litres and 900 litres. $10 \le x \le \frac{95}{4}$ $\mathbf{x \epsilon [\frac{1}{2}, \frac{5}{6}]}$	3
29	On her vacations Veena visits four cities A, B, C and D in a random order. What is the probability that she visits. (i) A before B? (ii) A before B and B before C? (iii) A first and B last? **Ans: 12/24 4/24 2/24**	3
30	Prove that : $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}=0$ **OR** Show that : $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}} = 2\cos\theta$	3

$$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\left(\frac{\frac{3\pi}{13}+\frac{5\pi}{13}}{2}\right)\cos\left(\frac{\frac{3\pi}{13}-\frac{5\pi}{13}}{2}\right)$$

$$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\frac{4\pi}{13}\cos\left(\frac{-\pi}{13}\right)$$

$$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\frac{4\pi}{13}\cos\frac{\pi}{13}$$

$$= 2\cos\frac{\pi}{13}\left[\cos\frac{9\pi}{13} + \cos\frac{4\pi}{13}\right]$$

$$= 2\cos\frac{\pi}{13}\left[2\cos\left(\frac{\frac{9\pi}{13}+\frac{4\pi}{13}}{2}\right)\cos\left(\frac{\frac{9\pi}{13}-\frac{4\pi}{13}}{2}\right)\right]$$

$$= 2\cos\frac{\pi}{13}\left[2\cos\frac{\pi}{2}\cos\frac{5\pi}{26}\right]$$

$$= 2\cos\frac{\pi}{13}\times 2\times 0\times\cos\frac{5\pi}{26}$$

$= 0 = RHS$

$$LHS = \sqrt{2+\sqrt{2+2\cos 4x}}$$

$$= \sqrt{2+\sqrt{2(1+\cos 4x)}}$$

$$= \sqrt{2+\sqrt{2\times 2\cos^2 2x}}$$

$$= \sqrt{2+2\cos 2x}$$

$$= \sqrt{2(1+\cos 2x)}$$

$$= \sqrt{2.2\cos^2 x}$$

$= 2\cos x = RHS$

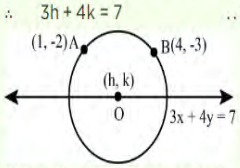
31 Find the equation of the circle passing through the points (1, –2) and (4, –3) and centre lies on the line 3x + 4y = 7. 3

OR

A rod of length 12 cm moves with its ends always touching the coordinate axes. Determine the equation of the locus of a point P on the rod, which is 3 cm from the end in contact x-axis.

Since, centre of circle lies on the line 3x + 4y = 7.

$\therefore \quad 3h + 4k = 7 \quad \ldots(i)$



Since, this circle passes through the points A(1, -2) and B(4, -3),

$\therefore$ OA = OB [radii of circle]

$\Rightarrow \quad OA^2 = OB^2$

$\Rightarrow \quad (1-h)^2 + (-2-k)^2 = (4-h)^2 + (-3-k)^2$

$\Rightarrow \quad 1 - 2h + h^2 + 4 + 4k + k^2 = 16 - 8h + h^2 + 9 + 6k + k^2$

$\Rightarrow \quad 6h - 2k = 20$

$\therefore \quad 3h - k = 10 \quad \ldots(ii)$

On solving Eqs. (i) and (ii), we get

$h = \frac{47}{15}$ and $k = -\frac{3}{5}$

So, the coordinates of centre of circle $\left(\frac{47}{15}, -\frac{3}{5}\right)$.

$\therefore$ Radius of circle = OA

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$$= \sqrt{\left(1-\frac{47}{15}\right)^2 + \left(-2+\frac{3}{5}\right)^2} = \sqrt{\left(-\frac{32}{15}\right)^2 + \left(\frac{-7}{5}\right)^2}$$

$$= \sqrt{\frac{1024}{225}+\frac{49}{25}} = \sqrt{\frac{1024+441}{225}} = \frac{\sqrt{1465}}{15}$$

So, the equation of circle having centre $\left(\frac{47}{15}, -\frac{3}{5}\right)$ and radius $\frac{\sqrt{1465}}{15}$ is

$$\left(x-\frac{47}{15}\right)^2 + \left(y+\frac{3}{5}\right)^2 = \left(\frac{\sqrt{1465}}{15}\right)^2$$

$$\therefore \quad \left(x-\frac{47}{15}\right)^2 + \left(y+\frac{3}{5}\right)^2 = \frac{1465}{225}$$

Let AB be the rod making an angle θ with positive direction of x-axis and P(x, y) be the point on it such that AP = 3cm

Now, PB = AB − AP = (12 − 3)cm = 9cm (AB = 12cm)

Draw PQ ⊥ OY and PR ⊥ OX

In △PBQ,

$$\cos\theta = \frac{PQ}{PB} = \frac{x}{9}$$

In △PRA,

$$\sin\theta = \frac{PR}{PA} = \frac{y}{3}$$

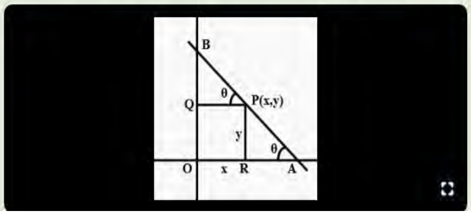
Since $\sin^2\theta + \cos^2\theta = 1$

Since $\sin^2\theta + \cos^2\theta = 1$

$$\Rightarrow \left(\frac{y}{3}\right)^2 + \left(\frac{x}{9}\right)^2 = 1$$

$$\Rightarrow \frac{x^2}{81} + \frac{y^2}{9} = 1$$

Thus the equation of the locus of point P on the rod is $\frac{x^2}{81} + \frac{y^2}{9} = 1$



SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each)

32 The mean and standard deviation of 20 observations are found to be 10 and 2 respectively. On rechecking, it was found that an observation 8 was incorrect. Calculate the correct mean and standard deviation if the wrong item is omitted. 5

OR

The following table gives the distribution of income of 100 families in a village. Calculate the mean and Standard Deviation:

Income (Rs)	0-1000	1000-2000	2000-3000	3000-4000	4000-5000	5000-6000
No. of Families	18	26	30	12	10	4

Mean = 10 $\Rightarrow \frac{\sum_{i=1}^{20} x_i}{20} = 10$

$\Rightarrow \sum_{i=1}^{20} x_i = 200$

SD = 2 $\Rightarrow \sigma^2 = 4$

$\Rightarrow \frac{\sum_{i=1}^{20} x_i^2}{20} - (10)^2 = 4$

$\Rightarrow \sum_{i=1}^{20} x_i^2 = 2080$

Thus, incorrect $(\sum_{i=1}^{20} x_i^2) = 200$ and incorrect $(\sum_{i=1}^{20} x_i^2) = 2080$

CASE (i) When the wrong item is omitted

On omitting 8, we are left with 19 observations.

$\therefore$ correct $(\sum_{i=1}^{19} x_i)$ = incorrect $(\sum_{i=1}^{20} x_i) - 8$

= (200 - 8) =192.

Thus, correct $(\sum_{i=1}^{19} x_i) = 192$

$\therefore$ correct mean = $\frac{192}{19} = 10.105 \ldots$ (i)

Also, correct $(\sum_{i=1}^{19} x_i^2)$ = incorrect $(\sum_{i=1}^{20} x_i^2) - 64$

= (2080 - 64) = 2016.

Mean=2320

Income	M	f (m-2500/1000)	fd	fd²
0 - 1000	18	-2	-36	72
1000-2000	1500	-1	-26	26
2000-3000	2500	0	0	0
3000-4000	350	+1	+12	12
4000-5000	4500	+2	+20	40
5000-6000	5500	+3	+12	36
		N = 100	$\sum fd = -18$	$\sum fd^2 = -186$

$$\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} \times 1000$$

$$= \sqrt{\frac{186}{100} - \left(\frac{-18}{100}\right)^2} \times 1000$$

$$= \sqrt{186 - 0.0324} \times 1000$$

$$= 1.3519 \times 1000$$

$$= \mathbf{1351.89}$$

33	a) Differentiate $sinx^2$ from first principle. b) If $y = \left(\frac{2-3\cos x}{\sin x}\right)$, find $\frac{dy}{dx}$ at $x = \frac{\pi}{6}$. c) Differentiate the following function w.r.t. x: $(x + \sec x)(x - \tan x)$ **OR** a) Differentiate tan(3x+1)from first principle b) Differentiate the following function w.r.t. x: (x + cot x) (x – cosecx) c) If for $f(x) = \lambda x^2 + \mu x + 12$, $f'(4) = 15$ and $f'(2) = 11$, then find λ and μ. **ANS: Using first principle find detivative** **12-4√3** $(x + \sec x)(1 - \sec^2 x) + (x - \tan x)(1 + \sec x \tan x)$ **OR** **a) Using first principle find derivative** **b) (x + cot x)(1+cosecx cotx)+ (x – cosecx)(1-**$cosec^2 x$**)** **c) λ=1, and μ=7.**	3+1+1
34	a) If the first and the nth terms of a G. P. are a and b respectively and if P is the product of the first n terms, prove that $P^2 = (ab)^n$. b) If $x = 1 + a + a^2 + \ldots\infty$, where $\|a\| < 1$ and $y = 1 + b + b^2 + \ldots\infty$, where $\|b\| < 1$. Prove that : $1 + ab + a^2b^2 + \ldots\infty = \frac{xy}{x+y-1}$ G.P. = $a, ar, ar^2 \ldots ar^{n-1}$ $\rightarrow b = ar^{n-1}$ $\rightarrow P = (a)(ar)(ar^2)\ldots(ar^{n-1})$ $\rightarrow P = a^n(1 \times r \times r^2 \times \ldots r^{n-1})$ $\rightarrow P = a^n r^{(1+2+3+\ldots+n+1)}$ $\rightarrow P = a^n r^{\frac{(n-1)n}{2}}$ Now, $\rightarrow P^2 = a^{2n} r^{n(n-1)}$ $\rightarrow P^2 = (a^2 r^{n-1})^n$ $\rightarrow P^2 = a^{2n} r^{n(n-1)}$ $\rightarrow P^2 = (a^2 r^{n-1})^n$ $\rightarrow P^2 = (a \cdot b)^n$ Hence $P^2 = (a \cdot b)^2$ $S_\infty = \frac{a}{1-r}$ Here, a = 1, r = a Therefore, $x = \frac{1}{1-a}$ $\Rightarrow 1 - a = \frac{1}{x}$ $\Rightarrow a = 1 - \frac{1}{x}$ $\Rightarrow y = 1 + b + b^2 + b^3 + \ldots$ $\Rightarrow a = 1, r = b$ $\Rightarrow y = \frac{1}{1-b}$ $\Rightarrow 1 - b = \frac{1}{y}$ $\Rightarrow b = 1 - \frac{1}{y}$ $1 + ab + a^2b^2 + a^3b^2 + \ldots$ $= 1 + ab + (ab)^2 + (ab)^2 + \ldots$ $= 1 + ab + (ab)^2 + (ab)^3 + \ldots$ $S_\infty = \frac{a}{1-r}$ Here a = 1, r = ab $= \frac{1}{1-ab}$ $= \frac{1}{1-\left(1-\frac{1}{x}\right)\left(1-\frac{1}{y}\right)}$ $= \frac{1}{1-\left(1-\frac{1}{y}-\frac{1}{x}+\frac{1}{xy}\right)}$ $= \frac{1}{1-1+\frac{1}{y}+\frac{1}{x}-\frac{1}{xy}}$ $= \frac{1}{\frac{1}{y}+\frac{1}{x}-\frac{1}{xy}} = \frac{1}{\frac{x+y-1}{xy}}$ $= \frac{1}{\frac{1}{y}+\frac{1}{x}-\frac{1}{xy}} = \frac{1}{\frac{x+y-1}{xy}}$ $= \frac{xy}{x+y-1}$	3+2
35	If $\tan\alpha = \frac{p}{q}$, where α = 6β, α being an acute angle, prove that	5

$$\frac{1}{2}\{p\,\mathrm{cosec}\,2\beta - q\sec 2\beta\} = \sqrt{p^2+q^2}$$

Since, we known that $\sin\theta = \frac{1}{\mathrm{cosec}\,\theta}$ and $\cos\theta = \frac{1}{\sec\theta}$, then we can write LHS of the above equation

$\frac{1}{2}\{p\,\mathrm{cosec}\,2\beta - q\sec 2\beta\} = \sqrt{p^2+q^2}$ as:

$$= \frac{1}{2}\left\{\frac{p}{\sin 2\beta} - \frac{q}{\cos 2\beta}\right\}$$

Take LCM of $\sin 2\beta$ and $\cos 2\beta$, then we will get:

$$= \frac{1}{2}\left\{\frac{p\cos 2\beta - q\sin 2\beta}{\sin 2\beta\cos 2\beta}\right\}$$

$$= \frac{p\cos 2\beta - q\sin 2\beta}{2\sin 2\beta\cos 2\beta}$$

We know that $\sin 2\theta = 2\sin\theta\cos\theta$, hence we will get

$$= \frac{p\cos 2\beta - q\sin 2\beta}{\sin 4\beta}$$

Now, we will multiply numerator and denominator with $\sqrt{p^2+q^2}$, we will get:

$$= \frac{\sqrt{p^2+q^2}}{\sin 4\beta}\left\{\frac{p\cos 2\beta - q\sin 2\beta}{\sqrt{p^2+q^2}}\right\}$$

Now, split $\sqrt{p^2+q^2}$ over both the numerator term:

$$= \frac{\sqrt{p^2+q^2}}{\sin 4\beta}\left\{\frac{p\cos 2\beta}{\sqrt{p^2+q^2}} - \frac{q\sin 2\beta}{\sqrt{p^2+q^2}}\right\}\text{............}(1)$$

Now, we will use triangle law of trigonometry (i.e. $\sin\theta = \frac{perpendicular}{hypotenuse}$, $\cos\theta = \frac{base}{hypotenuse}$ and $\tan\theta = \frac{perpendicular}{base}$)

It is given in question that $\tan\alpha = \frac{p}{q}$, hence perpendicular of the triangle is 'p' and its base is 'q', then, the

hypotenuse will become $\sqrt{p^2+q^2}$, so we can draw the below diagram:

$$= \frac{\sqrt{p^2+q^2}}{\sin 4\beta}\{\sin\alpha\cos 2\beta - \cos\alpha\sin 2\beta\}$$

Now, by using the formula $\sin(A-B) = \sin A\cos B - \sin B\cos A$, we can rewrite the above equation

as:

$$= \frac{\sqrt{p^2+q^2}}{\sin 4\beta}\sin(\alpha - 2\beta)$$

Now, we will put $\alpha = 6\beta$ in the above equation, then we will get:

$$= \frac{\sqrt{p^2+q^2}}{\sin 4\beta}\sin(6\beta - 2\beta)$$

$$= \frac{\sqrt{p^2+q^2}}{\sin 4\beta}\sin(4\beta)$$

$= \sqrt{p^2+q^2}$ = RHS

	SECTION E **CASE STUDY QUESTIONS**	
36	Alka is doing an experiment in which she has to arrange letters of word ALLAHABAD given in puzzle in order to form words with or without meaning using all letters a) How many words start and end with letter A? **1260** b) How many words can be formed when all A's donot come together? **7200** c) How many words have exactly 3 letters in between H and B. **1050**	1+2+1
37	In a game Ravi told his friend Mohan to make a 4-digit number greater than 5000 from the digits 0, 1, 3, 5 and 7 , then he asked him to calculate the Probabilty of forming number divisible by 5 when (i) the digits may be repeated **99/249** (ii) the repetition of digits is not allowed. **18/48**	2+2
38	A person is standing at a point **A** of a triangular park **ABC** whose vertices are **A**(2, 0), **B** (3, 4) and **C**(5, 6). Based on the above information answer the following :- a) Find the equation of **BC**. **x-y +1 = 0** b) Person **A** wants to reach on path **BC** in least time. Find the coordinates of the point on **BC** where he meets and the equation of the path he follows . **y+x =2 (1/2,3/2)** c) Find the shortest distance travelled by **A** to reach on path **BC**. **$3/\sqrt{2}$**	1+2+1