ITL PUBLIC SCHOOL
ANNUAL EXAMINATION(2023-24)

Date:16.02 .24 **Class: XI**

MATHEMATICS(041) – SET A(ANSWERKEY)

Time: 3 hrs **M. M: 80**

	SECTION A (Very Short Answers)	
1	Write the set A = $\{a_n : n \in N, a_{n+1} = 3a_n$ and $a_1 = 2\}$ in roster form **{2,6,18,54,….}**	1
2	Find the value of $\sin 15°$. **(√3 – 1)/(2√2)**	1
3	Solve for real values of x and y: $(1+i)y^2 + (6+i) = (2+i)x$ **x = 5, y = -2**	1
4	Let R be the relation on the set N of natural numbers defined by R = {(a, b): a + 3b = 12, a ∈ N, b ∈ N}. Find: (i) Domain of R (ii) Range of R **{9,6,3},{1,2,3}**	1
5	Solve the given inequality for real *x*: $\frac{5x-2}{3} - \frac{7x-3}{5} > \frac{x}{4}$ **(4,∞)**	1
6	Determine the number of 5 card combinations out of a deck of 52 cards if each selection of 5 cards has exactly one king. **4C1 x 48C4**	1
7	Calculate the sum of binomial coefficients in the expansion of $(1+x)^n$. **2^n**	1
8	Find the slope of the line, which passes through the origin, and the mid point of the line segment joining the points (0, - 4) and B(8, 0). **-1/2**	1
9	Find the length of the foot of the perpendicular from the point (3, 4, 5) on the y-axis. **√34**	1
10	Represent the solution of the given system of inequalities on a number line: $3x - 7 < 5 + x$; $11 - 5x \le 1$ **2 ≤ x < 6**	1
11	Find the lengths of the major and minor axes of the ellipse $9x^2 + 4y^2 = 36$. **6,4**	1
12	Evaluate: $\lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)}$ **1/π**	1
13	If $f(x) = 1 + x + \frac{x^2}{2} + ... + \frac{x^{100}}{100}$, then find $f'(1)$. **(100)**	1
14	Calculate the variance of the first 5 prime numbers. **mean = 5.6, Var = 10.24**	1
15	If the variance of the data 2, 4, 5, 6, 8, 17 is 23.33, then calculate the variance of 4, 8, 10, 12, 16, 34. **93.32**	1
16	A committee of two persons is selected from two men and two women. What is the probability that the committee will have one man? **2C1 x 2C1 / 4C2 = 2/3**	1
17	Tickets numbered from 1 to 20 are mixed up together and then a ticket is drawn at random. What is the probability that the ticket has a number which is a multiple of 3 or 7? **2/5**	1
18	If each term of an infinite GP is twice the sum of the terms following it, then find the common ratio of the GP. **1/3**	1
	ASSERTION-REASON BASED QUESTIONS (19,20) In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.	
19	ASSERTION: If 3x + 8 > 2, then x∈ { -1, 0, 1, 2, ….} when x is an Integer REASON: Solution set of the inequality 4x + 3 < 5x + 7 for all x∈R is [4, ∞). **C**	1
20	ASSERTION: The distance of the point P(x, y, z) from the origin (0, 0, 0) is given by $OP = \sqrt{x^2 + y^2 + z^2}$. REASON: If a point is on the x-axis then its y and z coordinates are 0. **B**	1

	SECTION B	
21	Prove that: $\dfrac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x} = \tan x$. $= \dfrac{2\sin 3x\cos 2x - 2\sin 3x}{-2\sin 3x \sin 2x} = -\dfrac{\sin 3x(\cos 2x - 1)}{\sin 3x \sin 2x}$ $= \dfrac{1-\cos 2x}{\sin 2x} = \dfrac{2\sin^2 x}{2\sin x\cos x} = \tan x = \text{R.H.S.}$	2
22	Reduce $\dfrac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-i\sqrt{2})}$ in standard form. **-7i/√2**	2
23	Using binomial theorem, prove that $6^n - 5n$ always leaves remainder 1 when divided by 25. $(1+5)^n = {}^nC_0 + {}^nC_1 5 + {}^nC_2 5^2 + \ldots + {}^nC_n 5^n$ i.e. $(6)^n = 1 + 5n + 5^2.{}^nC_2 + 5^3.{}^nC_3 + \ldots + 5^n$ i.e. $6^n - 5n = 1 + 5^2({}^nC_2 + {}^nC_3 5 + \ldots + 5^{n-2})$ or $6^n - 5n = 1 + 25({}^nC_2 + 5.{}^nC_3 + \ldots + 5^{n-2})$ or $6^n - 5n = 25k + 1$ where $k = {}^nC_2 + 5.{}^nC_3 + \ldots + 5^{n-2}$. This shows that when divided by 25, $6^n - 5n$ leaves remainder 1.	2
24	Evaluate: $\lim_{x\to 0}\dfrac{\sin 5x - \tan 2x}{3x - \sin^2 x}$ **1** **OR** Let *f(x)* be a function defined by $f(x) = \begin{cases} 6x-6, & x \le 3 \\ 2x-k, & x > 3 \end{cases}$, find k if $\lim_{x\to 3} f(x)$ exists. **k = - 6**	2
25	If E and Fare two events such that P(E) = $\frac{1}{4}$, P(F) = $\frac{1}{2}$ and P(E and F) = $\frac{1}{8}$ then find (i) P(E or F) (ii) P (not E and not F). **5/8, 3/8** **OR** A box contains 10 red marbles, 20 blue marbles and 30 green marbles. 5 marbles are drawn from the box, what is the probability that (i) all will be blue? (ii) at least one will be green? **20C5/60C5, 1-30C5/60C5**	2
	SECTION C	
26	(i) Find the domain and range of the real valued function *f* defined by $f(x) = \dfrac{1}{\sqrt{9-x^2}}$. Domain: (-3,3) Range: x = √(9y²-1) / y [1/3,∞) (ii) For sets A, B and C, using the properties of sets, prove that: $A-(B-C)=(A-B)\cup(A\cap C)$	2+1
27	A GP consists of even number of terms. If the sum of all the terms is 5 times the sum of the terms occupying the odd places, then find its common ratio.	3

	Let the G.P. be $T_1, T_2, T_3, T_4, \ldots T_{2n}$. Number of terms = $2n$ According to the given condition, $T_1 + T_2 + T_3 + \ldots + T_{2n} = 5\,[T_1 + T_3 + \ldots + T_{2n-1}]$ $T_1 + T_2 + T_3 + \ldots + T_{2n} - 5\,[T_1 + T_3 + \ldots + T_{2n-1}] = 0$ $T_2 + T_4 + \ldots + T_{2n} = 4\,[T_1 + T_3 + \ldots + T_{2n-1}]$ Let the G.P. be $a, ar, ar^2, ar^3, \ldots$ $\therefore \frac{ar(r^n-1)}{r-1} = \frac{4 \times a(r^n-1)}{r-1}$ $\Rightarrow ar = 4a$ $\Rightarrow r = 4$ **OR** The ratio of the AM and GM of two positive numbers *a* and *b* is *m : n*. Show that $a : b = \left(m + \sqrt{m^2 - n^2}\right) : \left(m - \sqrt{m^2 - n^2}\right)$ A.M $= \frac{a+b}{2}$ and G.M. = $\sqrt{ab}$ Apply componendo and dividend to get the result According to the given condition, $\frac{a+b}{2\sqrt{ab}} = \frac{m}{n}$	
28	Assuming that straight lines work as the plane mirror for a point, find the image of the point (1, 2) in the line $x - 3y + 4 = 0$. **(6/5, 7/5)** **OR** A person standing at the junction of two straight paths represented by the equations $2x - 3y + 4 = 0$ and $3x + 4y - 5 = 0$ wants to reach the path whose equation is $6x - 7y + 8 = 0$ in the least time. Find equation of the path that he should follow. **Point of intersection (-1/17, 22/17)** Slope of the line (3) $= \frac{6}{7}$ $\therefore$ Slope of the line perpendicular to line (3) $= -\frac{1}{\left(\frac{6}{7}\right)} = -\frac{7}{6}$ The equation of the line passing through and having a slope of $-\frac{7}{6}$ is given by $\left(y - \frac{22}{17}\right) = -\frac{7}{6}\left(x + \frac{1}{17}\right)$ $6(17y - 22) = -7(17x + 1)$ $102y - 132 = -119x - 7$ $119x + 102y = 125$ Hence, the path that the person should follow is $119x + 102y = 125$.	3
29	Prove that: $\cos 2\theta \cos\frac{\theta}{2} - \cos 3\theta \cos\frac{9\theta}{2} = \sin 5\theta \sin\frac{5\theta}{2}$	3

$$LHS = \frac{1}{2}\left[2\cos 2\theta \cos\frac{\theta}{2} - 2\cos 3\theta \cos\frac{9\theta}{2}\right]$$

$$LHS = \frac{1}{2}\,[\cos\{(2\theta+\theta/2)+\cos(2\theta-\theta/2)\} - \{\cos(3\theta+9\theta/2)+\cos(9\theta/2-3\theta)\}]$$

[Using: $2\cos A\cos B = \cos(A+B)+\cos(A-B)$]

$$LHS = \frac{1}{2}\left[\cos\frac{5\theta}{2}+\cos\frac{3\theta}{2}-\cos\frac{15\theta}{2}-\cos\frac{3\theta}{2}\right]$$

$$LHS = \frac{1}{2}\left[\cos\frac{5\theta}{2}-\cos\frac{15\theta}{2}\right]$$

$$LHS = \frac{1}{2}\left[2\sin\left(\frac{\frac{5\theta}{2}+\frac{15\theta}{2}}{2}\right)\sin\left(\frac{\frac{15\theta}{2}-\frac{5\theta}{2}}{2}\right)\right]$$

$$LHS = \sin 5\theta \sin\frac{5\theta}{2} = RHS$$

30 Find the derivative of $f(x) = x\sin x$ using first principle **Xcosx+sinx** — 3

31 Two students Anil and Ashima appeared in an examination. The probability that Anil will qualify the examination is 0.05 and that Ashima will qualify the examination is 0.10. The probability that both will qualify the examination is 0.02. Find the probability that: — 3

(i) both Anil and Ashima will not qualify the exam.
(ii) at least one of them will not qualify the exam.
(iii) only one of them will quality the exam.

(a) The event 'both Anil and Ashima will not qualify the examination' may be expressed as $E' \cap F'$.

Since, E' is 'not E', i.e., Anil will not qualify the examination and F' is 'not F', i.e. Ashima will not qualify the examination.

Also $E' \cap F' = (E \cup F)'$ (by Demorgan's Law)

Now $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

or $P(E \cup F) = 0.05 + 0.10 - 0.02 = 0.13$

Therefore $P(E' \cap F') = P(E \cup F)' = 1 - P(E \cup F) = 1 - 0.13 = 0.87$

(b) P (atleast one of them will not qualify)
$= 1 -$ P(both of them will qualify)
$= 1 - 0.02 = 0.98$

(c) The event only one of them will qualify the examination is same as the event either (Anil will qualify, and Ashima will not qualify) or (Anil will not qualify and Ashima will qualify) i.e., $E \cap F'$ or $E' \cap F$, where $E \cap F'$ and $E' \cap F$ are mutually exclusive.

Therefore, P(only one of them will qualify) $= P(E \cap F' \text{ or } E' \cap F)$

$= P(E \cap F') + P(E' \cap F) = P(E) - P(E \cap F) + P(F) - P(E \cap F)$

$= 0.05 - 0.02 + 0.10 - 0.02 = 0.11$

OR

On her vacations, Venna visits four cities (A, B, C and D) in a random order. What is the probability that she visits

(i) A before B (ii) A either first or second (iii) A first and B last.

S = {ABCD, ABDC, ACBD, ACDB, ADBC, ADCB
BACD, BADC, BDAC, BDCA, BCAD, BCDA
CABD, CADB, CBDA, CBAD, CDAB, CDBA
DABC, DACB, DBCA, DBAC, DCAB, DCBA}

(i) 1/2
(ii) 1/2
(iii) 1/12

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<tr><td></td><td>SECTION D</td><td></td></tr>
<tr><td>32</td><td>

If $\cos\theta = \dfrac{\cos\alpha\cos\beta}{1-\sin\alpha\sin\beta}$, prove that one value of $\tan\dfrac{\theta}{2} = \dfrac{\tan\frac{\alpha}{2} - \tan\frac{\beta}{2}}{1-\tan\frac{\alpha}{2}\tan\frac{\beta}{2}}$

$$\tan^2\frac{\theta}{2} = \frac{1-\cos\theta}{1+\cos\theta}$$

$$\tan^2\frac{\theta}{2} = \frac{1-\frac{\cos\alpha\cos\beta}{1-\sin\alpha\sin\beta}}{1+\frac{\cos\alpha\cos\beta}{1-\sin\alpha\sin\beta}}$$

$$\tan^2\frac{\theta}{2} = \frac{1-\sin\alpha\sin\beta-\cos\alpha\cos\beta}{1-\sin\alpha\sin\beta+\cos\alpha\cos\beta}$$

$$\tan^2\frac{\theta}{2} = \frac{1-(\cos\alpha\cos\beta+\sin\alpha\sin\beta)}{1+(\cos\alpha\cos\beta-\sin\alpha\sin\beta)}$$

$$\tan^2\frac{\theta}{2} = \frac{1-\cos(\alpha-\beta)}{1+\cos(\alpha+\beta)}$$

$$\tan^2\frac{\theta}{2} = \frac{2\sin^2\left(\frac{\alpha-\beta}{2}\right)}{2\cos^2\left(\frac{\alpha+\beta}{2}\right)}$$

$$\tan\frac{\theta}{2} = \pm\frac{\sin\left(\frac{\alpha-\beta}{2}\right)}{\cos\left(\frac{\alpha+\beta}{2}\right)}$$

$$\tan\frac{\theta}{2} = \pm\frac{\sin\frac{\alpha}{2}\cos\frac{\beta}{2}-\cos\frac{\alpha}{2}\sin\frac{\beta}{2}}{\cos\frac{\alpha}{2}\cos\frac{\beta}{2}-\sin\frac{\alpha}{2}\sin\frac{\beta}{2}}$$

$$\tan\frac{\theta}{2} = \pm\frac{\tan\frac{\alpha}{2}-\tan\frac{\beta}{2}}{1-\tan\frac{\alpha}{2}\tan\frac{\beta}{2}}$$

OR

If $\cos(\theta+\phi) = m\cos(\theta-\phi)$, then prove that $\tan\theta = \dfrac{1-m}{1+m}\cot\phi$

$$\Rightarrow \frac{\cos(\theta+\phi)}{\cos(\theta-\phi)} = \frac{m}{1}$$

Using componendo and dividendo theorem, we get

$$\frac{\cos(\theta+\phi)+\cos(\theta-\phi)}{\cos(\theta+\phi)-\cos(\theta-\phi)} = \frac{m+1}{m-1}$$

$$\Rightarrow \frac{2\cos\left(\frac{\theta+\phi+\theta-\phi}{2}\right).\cos\left(\frac{\theta+\phi-\theta+\phi}{2}\right)}{-2\sin\left(\frac{\theta+\phi+\theta-\phi}{2}\right).\sin\left(\frac{\theta+\phi-\theta+\phi}{2}\right)} = \frac{m+1}{m-1}$$

$$\Rightarrow \frac{\cos\theta.\cos\phi}{-\sin\theta.\sin\phi} = \frac{m+1}{m-1} \Rightarrow -\cot\theta.\cot\phi = \frac{m+1}{m-1}$$

$$\Rightarrow \frac{-\cot\phi}{\tan\theta} = \frac{m+1}{m-1} = -\frac{1+m}{1-m}$$

$$\Rightarrow \tan\theta = \frac{1-m}{1+m}\cot\phi.$$ Hence proved.

</td><td>5</td></tr>
<tr><td>33</td><td>

Show that the equation of the line passing through the origin and making an angle of θ with the line $y = mx + c$ is $\dfrac{y}{x} = \dfrac{m \pm \tan\theta}{1 \mp m\tan\theta}$.

</td><td>5</td></tr>
</table>

Let the equation of the line passing through the origin be $y = m_1 x$.

If this line makes an angle of θ with line $y = mx + c$, then angle θ is given by

$$\therefore \tan\theta = \left|\frac{m_1 - m}{1 + m_1 m}\right|$$

$$\Rightarrow \tan\theta = \left|\frac{\frac{y}{x} - m}{1 + \frac{y}{x}m}\right|$$

$$\Rightarrow \tan\theta = \pm\left(\frac{\frac{y}{x} - m}{1 + \frac{y}{x}m}\right)$$

$$\Rightarrow \tan\theta = \frac{\frac{y}{x} - m}{1 + \frac{y}{x}m} \text{ or } \tan\theta = -\left(\frac{\frac{y}{x} - m}{1 + \frac{y}{x}m}\right)$$

Case I: $\tan\theta = \dfrac{\frac{y}{x} - m}{1 + \frac{y}{x}m}$

$$\tan\theta = \frac{\frac{y}{x} - m}{1 + \frac{y}{x}m}$$

$$\Rightarrow \tan\theta + \frac{y}{x}m\tan\theta = \frac{y}{x} - m$$

$$\Rightarrow m + \tan\theta = \frac{y}{x}(1 - m\tan\theta)$$

$$\Rightarrow \frac{y}{x} = \frac{m + \tan\theta}{1 - m\tan\theta}$$

Case II: $\tan\theta = -\left(\dfrac{\frac{y}{x} - m}{1 + \frac{y}{x}m}\right)$

$$\tan\theta = -\left(\frac{\frac{y}{x} - m}{1 + \frac{y}{x}m}\right)$$

$$\Rightarrow \tan\theta + \frac{y}{x}m\tan\theta = -\frac{y}{x} + m$$

$$\Rightarrow \frac{y}{x}(1 + m\tan\theta) = m - \tan\theta$$

$$\Rightarrow \frac{y}{x} = \frac{m - \tan\theta}{1 + m\tan\theta}$$

Therefore, the required line is given by $\dfrac{y}{x} = \dfrac{m \pm \tan\theta}{1 \mp m\tan\theta}$.

34 Find the mean, variance and standard deviation: 5

Classes	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	7	12	15	8	3	2

Class	Frequency f_i	Mid-point x_i	$y_i = \frac{x_i - 65}{10}$	y_i^2	$f_i y_i$	$f_i y_i^2$
30-40	3	35	– 3	9	– 9	27
40-50	7	45	– 2	4	– 14	28
50-60	12	55	– 1	1	– 12	12
60-70	15	65	0	0	0	0
70-80	8	75	1	1	8	8
80-90	3	85	2	4	6	12
90-100	2	95	3	9	6	18
	N=50				– 15	105

Therefore $\bar{x} = A + \frac{\sum f_i y_i}{50} \times h = 65 - \frac{15}{50} \times 10 = 62$

Variance $\sigma^2 = \frac{h^2}{N^2}\left[N\Sigma f_i y_i^2 - (\Sigma f_i y_i)^2\right]$

$= \frac{(10)^2}{(50)^2}\left[50 \times 105 - (-15)^2\right]$

$= \frac{1}{25}[5250 - 225] = 201$

and standard deviation $(\sigma) = \sqrt{201} = 14.18$

OR

The mean and standard deviation of 100 observations were calculated as 40 and 5.1 respectively by a student who took by mistake 50 instead of 40 for one observation. Find the correct mean and standard deviation?

Solution Given that number of observations $(n) = 100$

Incorrect mean $(\bar{x}) = 40$,

Incorrect standard deviation $(\sigma) = 5.1$

We know that $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

i.e. $40 = \frac{1}{100}\sum_{i=1}^{100} x_i$ or $\sum_{i=1}^{100} x_i = 4000$

i.e. Incorrect sum of observations = 4000

Thus the correct sum of observations = Incorrect sum – 50 + 40

$= 4000 - 50 + 40 = 3990$

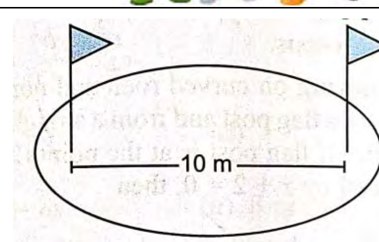
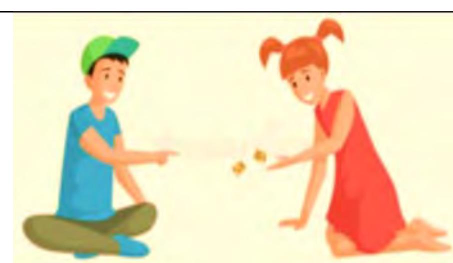
Hence Correct mean $= \frac{\text{correct sum}}{100} = \frac{3990}{100} = 39.9$

Also Standard deviation $\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \frac{1}{n^2}\left(\sum_{i=1}^{n} x_i\right)^2}$

$= \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - (\bar{x})^2}$

i.e. $5.1 = \sqrt{\frac{1}{100} \times \text{Incorrect} \sum_{i=1}^{n} x_i^2 - (40)^2}$

or $26.01 = \frac{1}{100} \times \text{Incorrect} \sum_{i=1}^{n} x_i^2 - 1600$

	Therefore Incorrect $\sum_{i=1}^{n} x_i^2 = 100\,(26.01 + 1600) = 162601$ Now Correct $\sum_{i=1}^{n} x_i^2 = \text{Incorrect} \sum_{i=1}^{n} x_i^2 - (50)^2 + (40)^2$ $= 162601 - 2500 + 1600 = 161701$ Therefore Correct standard deviation $= \sqrt{\dfrac{\text{Correct } \sum x_i^2}{n} - (\text{Correct mean})^2}$ $= \sqrt{\dfrac{161701}{100} - (39.9)^2}$ $= \sqrt{1617.01 - 1592.01} = \sqrt{25} = 5$	
35	Find the derivative of the following functions with respect to x: (i) $\dfrac{\sin x + \cos x}{\sin x - \cos x}$ (ii) $\dfrac{5x^3 - 4x^2 + 3}{\sqrt{x}}$ (iii) $x^2 \sin x + \sin^2 x$ **(i) $-2 / (\sin x - \cos x)^2$ (ii) $25/2\, x^{3/2} - 6\sqrt{x} - 3/2 x^{-3/2}$ (iii) $x^2\cos x + 2x\sin x + \sin 2x$**	2+2 +1
	SECTION E **CASE STUDY QUESTIONS**	
36	Out of 7 boys and 5 girls, a team of 7 students is to be formed. Use this data to answer the questions given below: (i) Find the number of ways if the team consists of at least 3 girls. (ii) If exactly 3 girls are selected and are arranged in a row for photograph, find the number of ways in which the girls and the boys can stand together (i) 10x35 + 5x35 + 1x21 = 546 (ii) Arranging: 2! x 3! x 4! = 2x6x24 =288 Selecting: 350 Required: 288 x 350 = 100800 	2+2
37	A man is running a race course such that the sum of distances of 2 flag posts from him is always 26m and the distance between the two flag posts is 10 m. Based on this information, find the : (i) equation of the path a=13, c=5, b=12 $x^2/169 + y^2/144 = 1$ (ii) eccentricity of the path 5/13 (iii) coordinates of the vertices of the path (±13,0) (iv) coordinates of the foci of the path (±5,0) 	4
38	A group of students were playing with a pair of dice and were trying to find chances for happening of a particular event. Use this data to find the probability of getting: (i) a doublet 1/6 (ii) two on at least one of the dice 11/36 (iii) odd numbers on both the dice 1/4 (iv) number 3 on one dice and 5 on the other. 1/18 	4